IAA AAI SECTION COLLOQUIUM 2019

THE MODERN ACTUARY - CHALLENGE • INFLUENCE • LEAD

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CTICC

Modeling of life reserving by using mixed methods

BALDE ALPHA – GUINEA

Hosted by

ACTUARIAL SOCIETY OF SOUTH AFRICA



About the speaker

BALDE ALPHA

- Senior consultant - Chief actuary of ACTIVA Guinea
- Qualified actuary within the morocan Association of Actuaries and the Guinean club of actuaries- Member of IAALS life section
- **I**nvolved in research topics (Mortality- lapses-Reserving-Machine Learning-Data Science)
- **A**uthor of papers in Microinsurance -**R**isk management-**S**tochastic modeling-SYSCO-OHADA and IFRS employees Benefits' Valuation

SECTION COLLOQUIUM 2019

About the Agenda

1. Motivation of the paper
2. Data set and processing rules
3. Loss reserves modeling

   3.1 Traditional Methods of life reserving

   3.2 Modeling life portfolio of death products and claims amount- Kaplan Meir AND weibull's family distribution
4. Theory of ruin for Risk margin
5. Findings And then ?

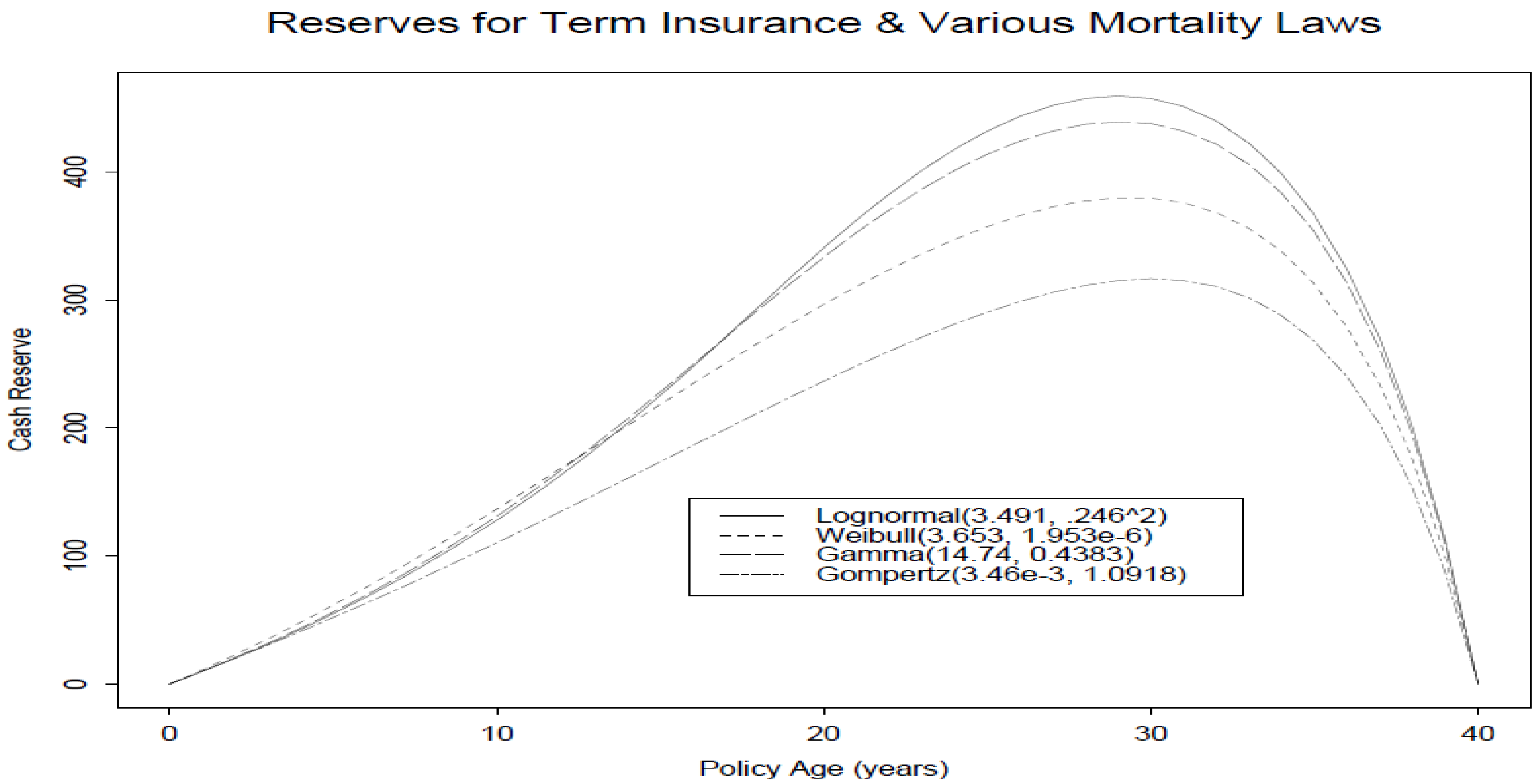
Motivation

- Using Actuarial standard reserving in life insurance (funeral cover; annuity, loans or banking account death insurance) is not easy with inconsistent data such as inappropriate mortality tables; errors on age policyholder; errors on capital insured ! Or significant number of missing values;

So in this context and without state-based regulation consideration ;

- **Could we view life insurance Predictive Modeling such as general insurance! ……if WE KNOW :** " **Reserve Liabilities for variable life insurance policies** shall be established consistent with the methodologies described in Standard Valuation Law of each regulation and in accordance with actuarial procedures that recognize **the variable nature of the benefits** provided and any **mortality guaran**tees" .

Actuarial Mathematics and Life-Table Statistics



Traditional life reserving -formulas

Equivalence Principle has the following definition:

Actuarial Present Value (APV) of Premiums equals to APV of Benefits plus APV of Charges.

Using a traditional endowment for a male aged x of duration n years, we have the following application

P''·äxn = Axn +α+γ·äxn

äxn = the net single premium for a n-year temporary life annuity-due which provides for annual payments of 1 unit as long as the beneficiary lives.

Axn = the net single premium for an endowment which provides for a payment of 1 unit at the end of the year of death if it occurs within the n first years otherwise at the end of the nth year.

PM_k = $VAP(INSURANCE)_k$ - $VAP(\text{ insured})_k$ at the end year k

FORMULES DE CALCUL DES P.M. inventaire (pour C frs de capital garanti)

n = durée contrat ; p = durée paiement primes ; x = âge assuré ; k= k^{ième} anniversaire contrat(date calcul PM)
PI = prime annuelle d'inventaire pour 1 franc de capital garanti

TC	CHARGT	CONTRATS A PRIME UNIQUE	CONTRAT A PRIME ANNUELLE
CD	g1=3. 5°/°° g2=1. 5°/°° f = 8%	$C * \frac{D_{x+n} + 0.0015*(N_{x+k} - N_{x+n})}{D_{x+k}}$	$k < p$ $C*[\frac{D_{x+n} + 0.0015*(N_{x+k} - N_{x+n}) + (0.0035 - PI)*(N_{x+k} - N_{x+p})}{D_{x+k}}]$ $k \geq p$ $C*[\frac{D_{x+n} + 0.0015*(N_{x+k} - N_{x+n})}{D_{x+k}}]$
T D	g1= 0 g2=1°/°°	$M_{x+k} - M_{x+n} + 0.001*(N_{x+k} - N_{x+n})$ C * ---	$k < p$ $C * [\frac{M_{x+k} - M_{x+n} + 0.001*(N_{x+k} - N_{x+n}) - PI*(N_{x+k} - N_{x+p})}{D_{x+k}}]$ $k \geq p$

Global approach **modeling portfolio Premium liability**

How to Substitute Traditionnal life reserving unit policyholder/policyholder by the Global Reserving of Portfolio of the same pooling risk…..

FIRST ***step 1:*** *modeling* the average mean lifetime of portfolio in months ;

then ***step 2:*** *modeling* the monthly expected amount of claims and estimating the total current claims on the lifetime duration of portfolio valuated *on step1* with the appropriate discount rate or yield curve;
We assume both the **uniformity of monthly cash flows of claims** during the coverage period and that the period until settlement is similar for all claims.

dataset of life claims - garantees and premiums of the company

Date of claim occurrence : not paid claims but occured claims for life reserving.

Flow of Claims File by MONTH

Date de Surver	Date Effet	Date Expiratior	Type d'Evaluat	Montant à pay	average life time	Produit
13/08/2012	01/01/2012	31/12/2012	Décès (Mig)	6 727 202	8	ASSURANCE DES COMPTES (REGU
11/01/2017	25/02/2012	24/02/2017	Décès	3 750 000	60	ASSURANCE DES COMPTES (REGU
11/01/2016	24/03/2012	23/03/2017	Décès	20 000 016	47	ASSURANCE DES COMPTES (REGU
13/10/2014	21/05/2012	20/05/2016	Décès	11 443 666	30	ASSURANCE DES COMPTES (REGU
14/10/2016	17/07/2012	16/07/2017	Décès	5 400 000	52	ASSURANCE DES COMPTES (REGU
17/03/2016	25/07/2012	24/07/2017	Décès	6 900 014	45	ASSURANCE DES COMPTES (REGU
24/08/2016	17/08/2012	16/08/2017	Décès	15 080 836	49	ASSURANCE DES COMPTES (REGU
19/05/2017	25/08/2012	24/08/2017	Décès	2 500 000	58	ASSURANCE DES COMPTES
27/09/2016	27/08/2012	26/08/2017	Décès	2 894 354	50	ASSURANCE DES COMPTES (REGU
10/08/2016	08/11/2012	07/11/2016	Décès	69 854 813	46	GROUP EMPRUNTEURS
11/07/2015	01/01/2013	31/12/2017	Décès	42 666 676	31	GROUP EMPRUNTEURS
09/12/2017	01/01/2013	31/12/2017	Décès	1 216 647	61	GROUP EMPRUNTEURS

Flow of Premiums File by Month

ID Assurée	Nom Assurée	Frais	Taxes	Prime Nette	Prime TTC	CA	Commission
62435	BICIGUI	20000	6640270	132785400	139445670	132 805 400	13278540
108934	TOTAL-GUINEE	20000	18526181	370503545	389049723	370 523 545	37050354,5
102214	SOMCAG	20000	1070234	21384715	22474949	21 404 715	2138471,5
129635	FBN BANK	20000	370255	7384482	7774737	7 404 482	738448,2
113680	UGAR-ACTIVA	20000	1501225	30004500	31525725	30 024 500	0
114523	ACTIVA-VIE	20000	655000	13080000	13755000	13 100 000	0
17861	PLAN-GUINEE	20000	20565264,14	411285282,8	431870547	411 305 283	41128528,28
151968	LANALA .	0	565005	11299995	11865000	11 299 995	0
158790	AUDITEURS ASSOCIES EN AFRIQUE GUINEE	300000	953565	18771435	20025000	19 071 435	1877143,5

Controls on the data Rules Processing for global approach reserving

- **Constitution of the database: a priori controls Checking**

The objective is to determine the events that can, a priori, impact the quality of the database or conduct to incomplete data:

- **Portfolio Repurchase**
- **Entities merging**
- **management system changes**
- **evolution of management processes**
- **etc.**

- **Constitution of the database: Internal consistency checks**

These checks are intended to ensure the internal consistency of the database created. They carry at least on:

- **Exhaustivity and independence of data**
- **The correct chronology of the dates in the individual data**
- **The likelihood of the information contained in the** databases (premiums related to guarantees, etc.); accounting elements (services, number of contracts, technical accounts, etc.); Previous financial audit reports; **dashboard of monthly Production**; **dashboard** of monthly **CLAIMS** , **Ratemaking** etc.

Modeling Life time Portfolio Kaplan Meir

Step 1:

The Kaplan-Meier method provides a quick survival curve, as well as essential statistics such as **the median or mean residual survival time** . The Kaplan-Meier method makes it possible to estimate the survival functions, without requiring that the time intervals be regular, unlike the actuarial survival tables method.

Kaplan Meier and survival models are adapted for modeling Death risk and life time of life Insurance contracts ;

Modeling Life time Products Kaplan Meir Estimator

ASSUMPTIONS

It is supposed that there are no censors in the observation of the survival time insured population at risk from cohort from **2012 to 2018.** We consider as data surrounding **"Event giving benefits or loss :**

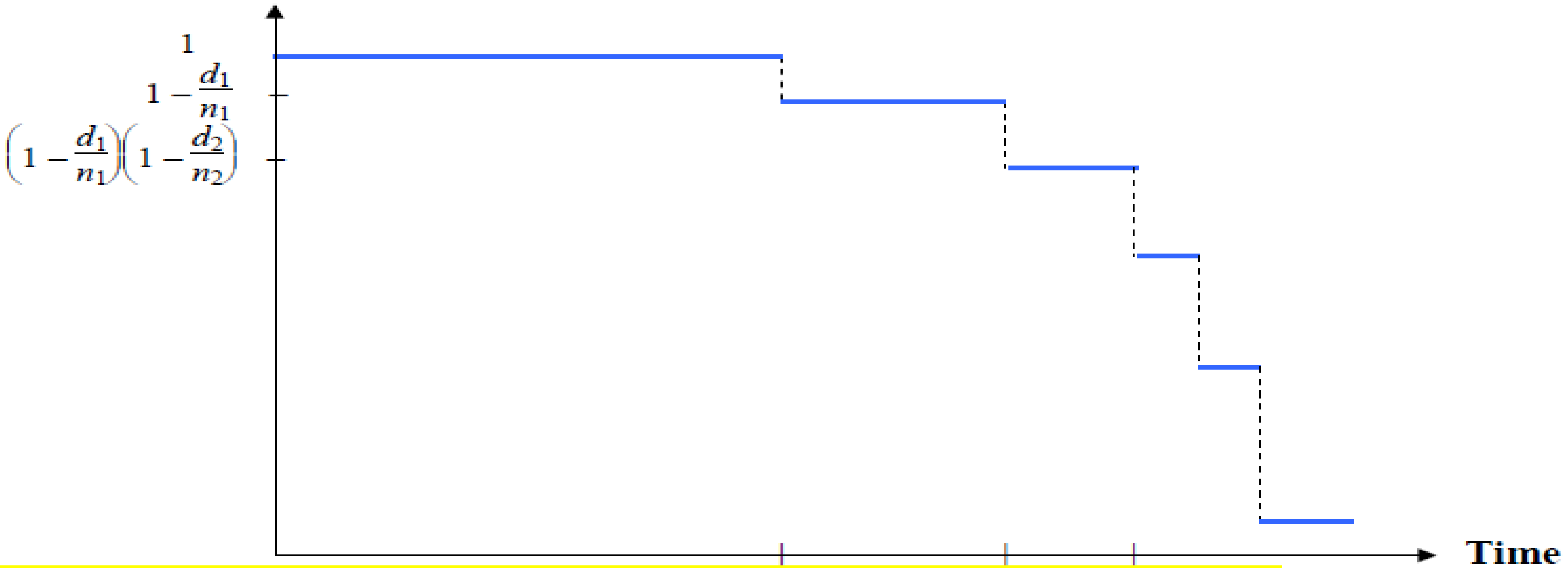
- **The death of the insured;**
- **Loss of employment of the policyholder loan;**
- **Maturity or life -term of the contract**.

For any time $t \in [0, t1)$, we have **$S(t) = P(T > t)$ = "Probability of surviving beyond time t,**

This is known as the Kaplan-Meier estimator of the survival function S(t). (Theory
developed in 1950s, but first implemented with computers in 1970s.) Note that it is not continuous, but only piecewise-continuous (actually, piecewise-constant, or "step
function"). **In general, for $t \in [tj, tj+1)$, j = 1, 2, 3,we have……………….;**

Modeling Life time Products Kaplan Meir Estimator

$$\hat{S}(t) = \left(1-\frac{d_1}{n_1}\right)\left(1-\frac{d_2}{n_2}\right)\cdots\left(1-\frac{d_j}{n_j}\right) = \prod_{i=1}^{j}\left(1-\frac{d_i}{n_i}\right).$$



times of death:	0	t_1	t_2	t_3	...
# deaths:	0	d_1	d_2	d_3	...
# survivors:	$n_1 = n - 0$	$n_2 = n_1 - d_1$	$n_3 = n_2 - d_2$		...

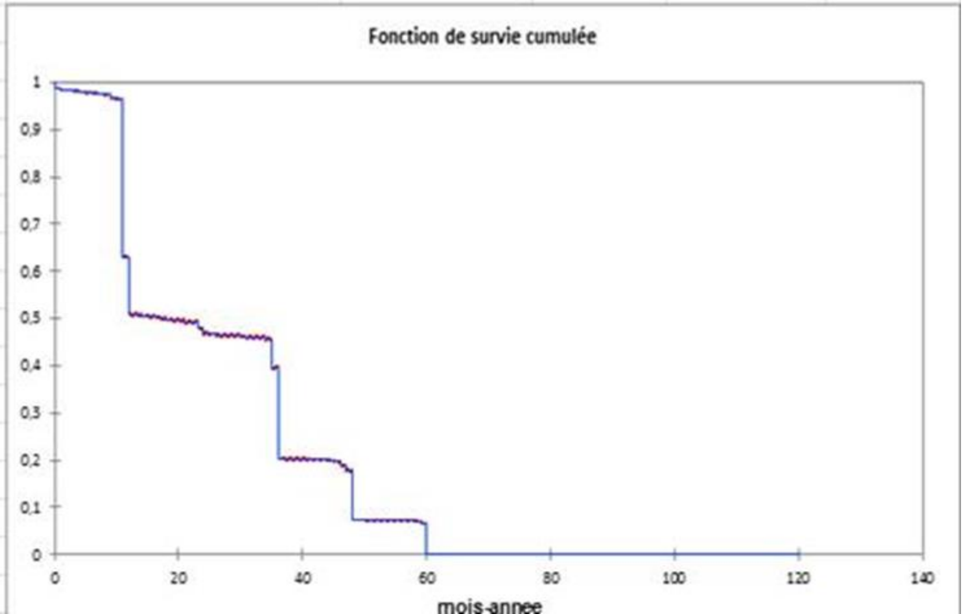
Modeling Average life survival of portfolio-KAPLAN MEIR –XLSTAT

Temps de survie moyen :

Temps de survie moyen	Ecart-type	e inférieure (	ie supérieure (95%)
26,114	0,060	25,996	26,233

Estimation des quantiles :

Quantile	Valeur estimée	e inférieure (	ie supérieure (95%)
75%	36,000	0,000	120,000
50%	17,500	16,000	19,000
25%	11,000	0,000	120,000



Modeling Average life amount claim -XLSTAT

step 2 : Most data in general insurance is skewed to the right and therefore most distributions that exhibit this characteristic can be used to model the claim amount and will concerned with modelling claim amounts by fitting probability distributions from selected families to set on observed claim sizes. Steps of modelling process follow as below:

- We will assume that the claims arise as realizations from a certain family of distributions after an exploratory analysis and graphical techniques. The distributions used in this article include **gamma**, **Weibull or Exponential , lognormal** and **Pareto:**

  Exponential : $P(X > x) = \exp(-\theta x)$; Weibull : $P(X > x) = \exp(-\theta x^\gamma)$;
- We will estimate the parameters of **the selected parametric distribution** using maximum likelihood based on the claim amount records: Historical experience predict future;
- We will test whether the selected distribution provides an adequate fit to the data using **Kolmogorov-Smirnov, Anderson-Darling or χ^2 test** or Q-Q plot,

Modeling Average life amount claim WEIBULL FAMILY –XLSTAT

Synthèse de l'ajustement automatique :

Distribution	p-value
Binomiale ne	0
Binomiale ne	0,281
Khi²	0
Fisher-Tippe	0
Fisher-Tippe	0,011
Gamma (1)	0
Gamma (2)	0,548
GEV	0,009
Gumbel	0
Log-normale	0,271
Logistique	0,038
Normale	0,016
Normale sta	0
Poisson	0
Weibull (1)	0
Weibull (2)	**0,868**

La distribution qui s'ajuste le mieux aux données pour le test d'ajust

Paramètres estimés(Weibull (2)) :

Paramètre	Valeur	Erreur standard
bêta	0,733	
gamma	143928401	4,277

Statistiques de Log-vraisemblance :

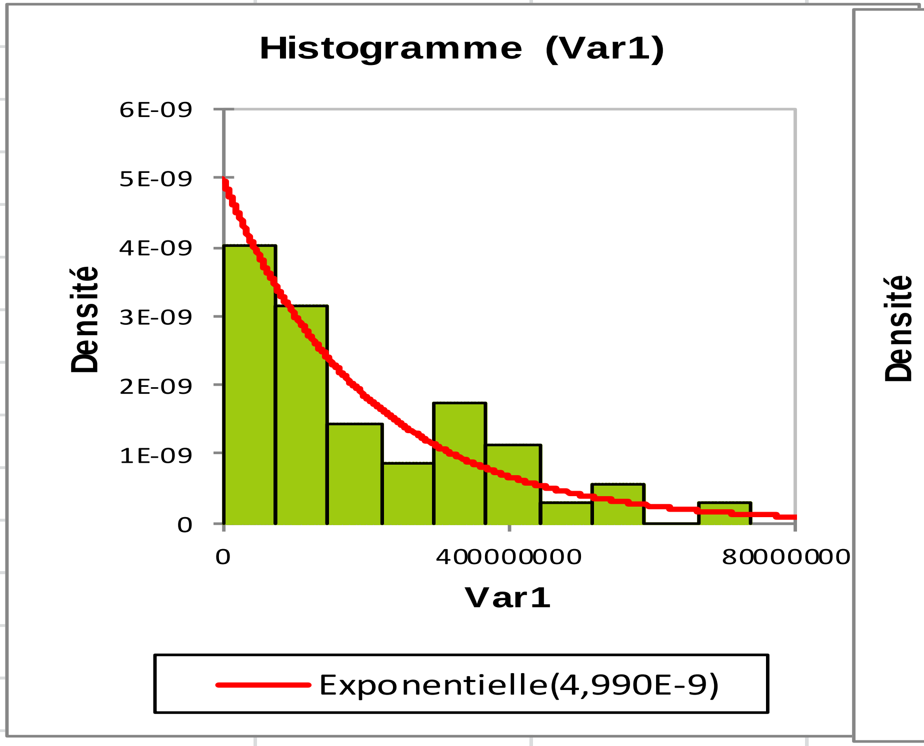
Log-vraiseml	-773,766
BIC(LV)	1554,858
AIC(LV)	1551,531

Statistiques estimées à partir des données et calculées à partir des e

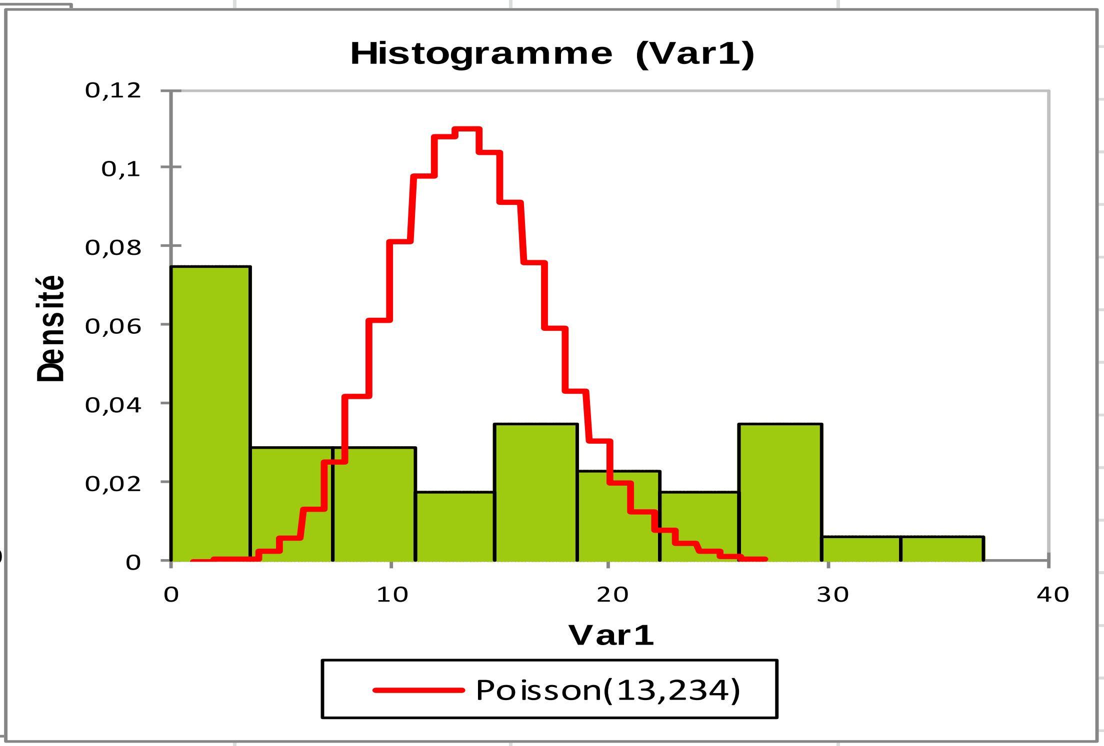
Statistique	Données	Paramètres

Distribution of claims –Parameters by using Non life Methods-frequency and amount

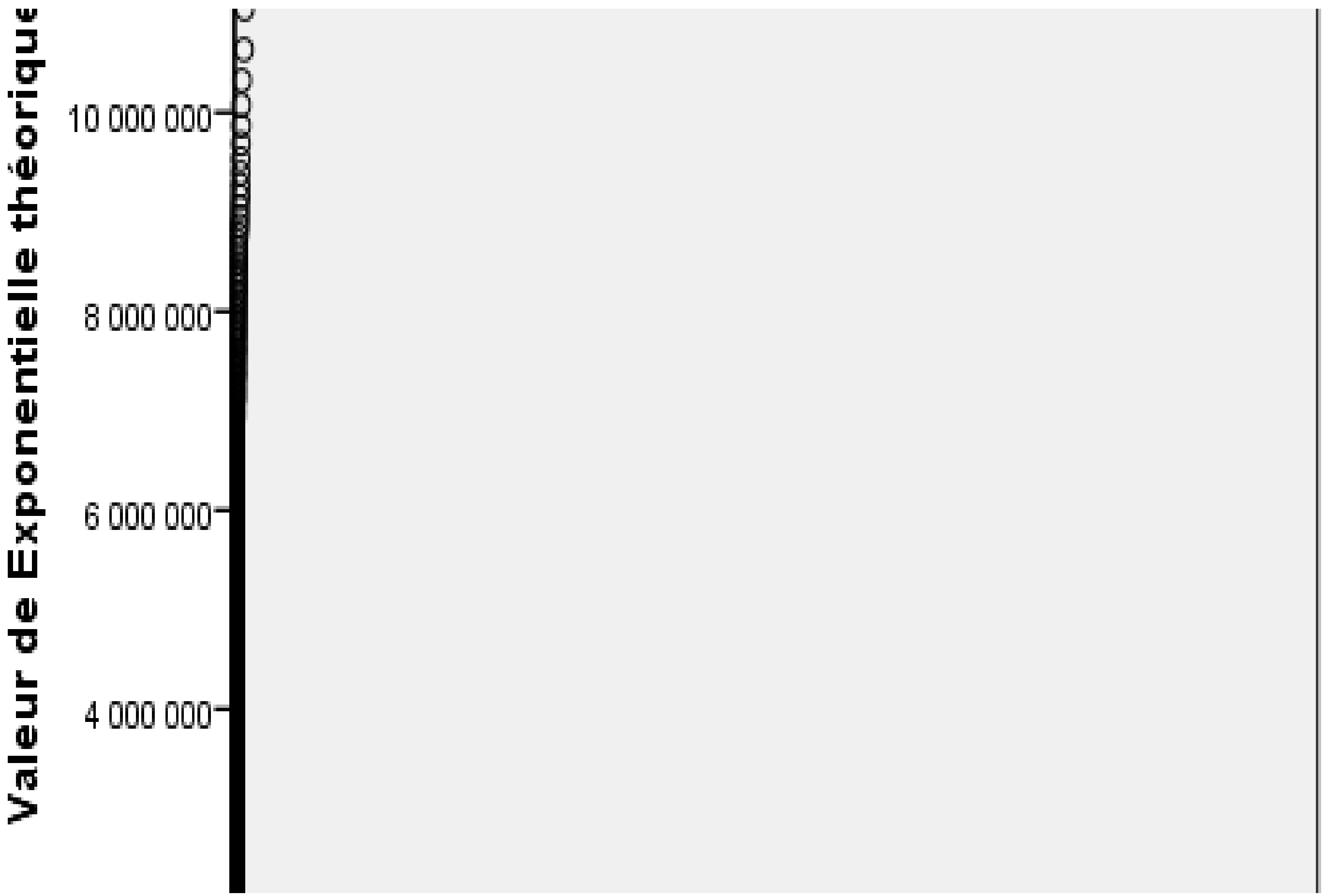
modeling monthly of claims



Modeling monthly number of claims



Distribution of premiums amount-QQ Plot



Initial Reserve for Life best Estimate Reserving !

- **Annual discount rate i=3,5/100 (traditionnel rate used within CIMA African Insurance companies regulator),**

  Let us note :
- **u= initial réserve**
- **µ= average MONTHLY amount of claims (weibull-exponantial estimator)**
- **n= average mean life time of portfolio(Kaplan Meier estimator)**
- We assume both the **uniformity of monthly cash flows of claims** during the coverage period and that the period until settlement is similar for all claims.

- **we have Current estimate initial Reserve as the sum of geometrical suit expressed below:**

U= µ*(1- Puissance(q; n))/(1-q) with q = monthly Equivalence of actualization factor q =1/(PUISSANCE((1+i), 1/12)),

But The <u>Best estimate Reserving = u + Risk margin</u>

Risk theory model For reserving

- **INFLOW= premiums**

✓ – deterministic
– steady
– can be forecasted

- **OUTFLOW=claims**

– stochastic, i. e. of random nature

– subject to fluctuations
– cannot be forecasted easily

- **LEVEL at time t of cash Reserving**
  = initial reserves + accumulated premiums – accumulated claims

Ruin theory for Life best Estimate Reserving !

Cramer-Lundberg Classic Model assumptions

- The company's portfolio is simple, meaning that the company does not only one kind of insurance.
- Claims form a parameter Poisson process ,

$$P(N(t)=k)=\frac{(\lambda t)^{\kappa}}{k!}e^{-\lambda t},\quad k=\overline{0:\infty},\qquad t\geq 0$$

- Initial capital (or reserve) is equal to z (Currency Units).
- Premiums arrive at the checkout on a regular and deterministic basis so that capital received until time t is equal to ct (c UM = time unit).

Ruin theory for Life best Estimate Reserving !

Cramer-Lundberg Classic Model assumptions

- The consecutive indemnification amounts (paid momentarily at the claims: 0 X1; X2; X3;……X(t)) are independent random variables and independent of the POISSON process N (t). They all follow the same distribution law F (x).

$$Y_z(t) = z + ct - \sum_{k=0}^{N(t)} X_k$$

Let us denote by **Yz (t)** the reserve which is in the cash register of the company at **time t.**

How to choose good Risk margin!

Average life time portfolio	30,	Months
Technical discount rate	3,50%	

Assumptions of Basic simulation

- u = initial reserve = 5 757 474 418 GNF
- t = time unit in months
- c = monthly premium rate = 2 681 741 260 GNF (estimation based from exercises 2012-2018)
- λ = average monthly number of claims, **13** CLAIMS BY MONTH
- µ = average MONTHLY amount of claims 2 00 000 000 GNF
- δ = parameter of EXPONENTIONAL Distribution = 4.99 E-9

simulations-Reserves ruine theory

Loading on Reserve	ruin probability	RESERVE Adjusted
0%	40%	5 757 474 418,00
5%	39%	6 045 348 138,90
10%	37%	6 333 221 859,80
15%	35%	6 621 095 580,70
20%	34%	6 908 969 301,60
25%	32%	7 196 843 022,50
30%	31%	7 484 716 743,40
35%	30%	7 772 590 464,30
40%	28%	8 060 464 185,20
45%	27%	8 348 337 906,10
50%	26%	8 636 211 627,00
55%	25%	8 924 085 347,90
60%	24%	9 211 959 068,80
65%	23%	9 499 832 789,70
70%	22%	9 787 706 510,60
75%	21%	10 075 580 231,50
80%	20%	10 363 453 952,40
85%	19%	10 651 327 673,30
90%	18%	10 939 201 394,20
95%	18%	11 227 075 115,10
100%	17%	11 514 948 836,00
105%	16%	11 802 822 556,90
110%	15%	12 090 696 277,80
115%	15%	12 378 569 998,70

Findings –And Then

- In the context of inconsistent life data - inadequate mortality tables and low interest rates ; Modeling life insurance such as general insurance could ***give strong opinions in Solvency to the actuary in africa*** .
- For the regulators, premium liability methodology may only be permissible if it can be demonstrated to be a sufficiently reliable approximation of the present value of expected risk-weighted future cash flows .(**IAA LIABILIBITIES-Measur**ement)
-

we   must   also explore   :

- Using   **Times series Analysis –Box jenkins** for modeling et **Forecasting monthy Loss distributions occurrence trough the life time of portfolio**
- Using   **Copulas** and **levy processus** method for Modeling **Ruin Probability**
- **Using GLM in life insurance for modeling the probability of death-surrending-lapse relating to year exposure;duration or category of contract**

End

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Thanks you for your attention!

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