

A new Notional Defined Contribution system based on risk sharing

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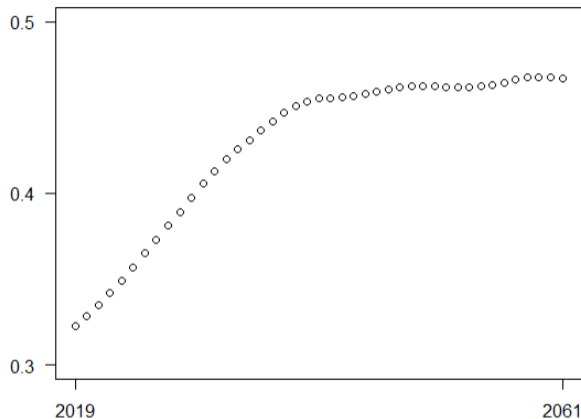
4 April 2019

2019 IAA Colloquium
Cape Town – South Africa

Ageing population

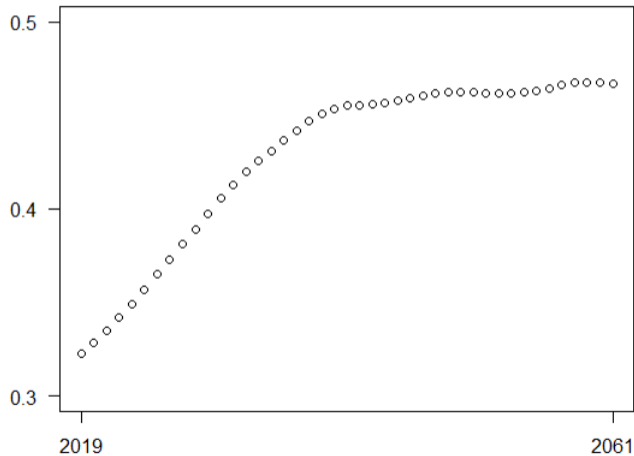
- Increasing life expectancy
- Decreasing birth rate
- *Papy boom*

Dependence ratio



Federal Planning Bureau, 2016

Dependence ratio



Defined benefit system

45% increasing
in contributions

Literature review

- Traditional DB system in **crisis**

Alonso-García, Boado-Penas, Devolder, 2017

- A **solution** with automatic adjustments : Notional Defined Contribution system

Palmer, 2000 ; Holzmann, Palmer, Robalino, 2012

Settergren 2001 ; Alonso-García, Boado-Penas, Devolder, 2018

- The NDC system and its automatic adjustments lead to a **dramatic reduce** of the pension benefits

OECD, 2017 ; OECD, 2018

- Our alternative proposition : adapted NDC system with risk sharing through **hybrid pension plans**

Musgrave, 1981

Outline

Notional Defined Contribution scheme

Hybrid schemes

Our adapted NDC scheme

Risk sharing

Numerical application

Notional Defined Contribution scheme

Classical NDC system

Automatic adjustments

Notional Defined Contribution system

The Notional Defined Contribution scheme is a **PAYG mechanism** mimicking a fully funded system with **defined contributions** and based on individual notional accounts.

Constitution of the notional capital

The individual notional capital accumulated during the career is

$$C_t = \pi \sum_{m=1}^{x_r - x_0} S_{t-m} \prod_{k=1}^m (1 + r_{t-k})$$

with

- π the contribution rate
- S the salary
- r the notional rate

Conversion of the notional capital

First pension benefits when entrance into retirement are

$$P_t = \frac{C_t}{a_{x_r,t}}$$

with the the life annuity

$$a_{x_r,t} = \sum_{m=0}^{\omega-x_r} m p_{x_r,t} (1 + \hat{g}_t)^m (1 + \hat{r}_t)^{-m}$$

and

- p_x the survival probability (generational life table)
- \hat{g} the projected pension indexation rate
- \hat{r} the projected notional rate

Automatic adjustments

In the Swedish system

In the aim to maintain the financial sustainability of the system, automatic balance mechanisms based on demographic and economic variables are implemented.

1. The criterion is

$$\text{balance ratio} = \frac{\text{contribution asset} + \text{buffer fund}}{\text{pension liability}} < 1$$

2. The adjustment variables are

- the notional rate
- the benefits indexation rate

Automatic adjustments

In the current ageing context

The DC environment **protects the contributions** of the workers and the usual automatic adjustments impact only the benefits of the current and future retirees.

In this ageing context, the replacement rate is projected to **dramatically decrease**.

Hybrid schemes

Pay As You Go system

Automatic adjustments

Specific schemes

PAYG scheme

The equilibrium equation of the PAYG scheme is

$$\begin{aligned}\text{Contributions} &= \text{Benefits} \\ W_t \pi_t \bar{S}_t &= R_t \bar{P}_t \\ &= R_t \delta_t \bar{S}_t\end{aligned}$$

with

- W : number of workers
- π : contribution rate
- \bar{S} : mean salary
- R : number of retirees
- δ : benefit ratio
- \bar{P} : mean pension

PAYG equilibrium equation

The equilibrium equation of the PAYG scheme is

Contributions = Benefits

$$W_t \pi_t \bar{S}_t = R_t \delta_t \bar{S}_t$$

$$\pi_t = D_t \delta_t$$

with the dependence ratio

$$D_t = \frac{\# \text{ retirees}}{\# \text{ workers}}$$

PAYG equilibrium equation

The equilibrium equation of the PAYG scheme is

$$\begin{aligned}\text{Contributions} &= \text{Benefits} \\ W_t \pi_t \bar{S}_t &= R_t \delta_t \bar{S}_t \\ \pi_t &= D_t \delta_t\end{aligned}$$

with the dependence ratio

$$D_t = \frac{\# \text{ retirees}}{\# \text{ workers}} \quad \text{risk factor}$$

Automatic balance mechanism

In case of increase of the risk factor D , how can

- the contribution rate π
- the benefit ratio δ

be **automatically adjusted** to maintain the **equilibrium** ...

... while maintaining simultaneously financial sustainability and social adequacy?

DB and DC schemes

Defined Benefit (DB)

δ constant

$$\pi_t = D_t \delta$$

Demographic risk borne
by the **workers**

Defined Contribution (DC)

π constant

$$\delta_t = \frac{\pi}{D_t}$$

Demographic risk borne
by the **retirees**

A hybrid scheme

With the traditional DB and DC schemes, the demographic risk is entirely borne by either the workers or the retirees.

Hybrid system shares this risk between both generations.

One example of hybrid system is the [Musgrave rule](#).

The Musgrave rule

Constant benefit ratio **net of contribution**

$$M = \frac{\bar{P}_t}{\bar{S}_t(1 - \pi_t)} = \frac{\delta_t}{1 - \pi_t} \quad \left\{ \begin{array}{l} \delta_t = \frac{M}{1 + M D_t} \\ \pi_t = \frac{M D_t}{1 + M D_t} \end{array} \right.$$

Demographic risk **shared** by the workers and the retirees

Our adapted Notional Defined Contribution scheme

Automatic adjustments of the contribution

Automatic adjustments of the benefits

PAYG equilibrium

Why a new NDC system ?

With the classical NDC system, the demographic and economic risks are entirely borne by the benefits of the current and future retirees.

In the aim to share the risk between all generations : the workers through their contributions and the retirees through their benefits, we propose an **adapted NDC system** :

a hybrid scheme following the principles of the NDC system.

Adapted NDC system

Adjustment of the contributions

The contribution rate becomes¹

$$\pi_t = \tilde{\pi} (1 + \lambda_t)$$

with

- π the real contribution rate (paid)
- $\tilde{\pi}$ the notional contribution rate (benefits computation)
- λ the contribution adjustment rate

1. By analogy with the *taux d'appel des cotisations* in the French points system (AGIRC-ARRCO)

Adapted NDC system

Adjustment of the benefits

After retirement, individual pension benefits evolve as

$$\tilde{P}_{t+\Delta} = P_t \frac{\bar{S}_{t+\Delta}}{\bar{S}_t} \beta_t^\Delta$$

with $\beta \leq 1$ the cumulative sustainability rate.

The benefit ratio is a weighted mean of the current pension benefits

$$\delta_t = \frac{1}{\bar{S}_t} \int_{x_r}^{\omega} P_{t-(x-x_r)} \beta_{t-(x-x_r)}^{x-x_r} l_{x,t} dx$$

with

- l the population density
- x_r the retirement age
- ω the oldest age in the population

PAYG equilibrium

Within our adapted NDC system

The contribution rate and the benefit ratio are adapted such that the **PAYG equilibrium** is maintained

$$\pi_t = D_t \delta_t$$

with

$$\begin{cases} \pi_t = \tilde{\pi} (1 + \lambda_t) \\ \delta_t = \frac{1}{S_t} \int_{x_r}^{\omega} P_{t-(x-x_r)} \beta_{t-(x-x_r)}^{x-x_r} l_{x,t} dx \end{cases}$$

PAYG equilibrium

Within our adapted NDC system

3 equations and 4 adjustment variables

$$\begin{cases} \pi_t = D_t \delta_t \\ \pi_t = \tilde{\pi} (1 + \lambda_t) \\ \delta_t = \frac{1}{S_t} \int_{x_r}^{\omega} P_{t-(x-x_r)} \beta_{t-(x-x_r)}^{x-x_r} l_{x,t} dx \end{cases}$$

We need an additional constraint.

An additional constraint

The additional constraint can be

- π constant for the DC (or NDC) system
- δ constant for the DB system
- $\frac{\delta}{1-\pi}$ constant for the Musgrave rule

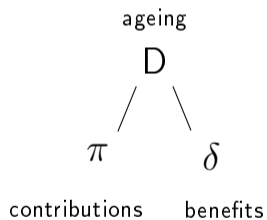
Or any other relation between the contribution rate π and the benefit ratio δ .

Risk sharing between workers and retirees

Optimal intermediate

Within our adapted NDC system

Risk sharing between workers and retirees



The system following the Musgrave rule is a possible hybrid system.

Is it optimal?

Our optimization

A possible loss function to minimize is

$$(1 - \rho) \left(\frac{\delta_t}{\bar{\delta}} - 1 \right)^2 + \rho \left(\frac{\pi_t}{\bar{\pi}} - 1 \right)^2$$

with fix targets $\bar{\delta}$, $\bar{\pi}$ and a constant given weight parameter $\rho \in [0, 1]$.

Cairns, 2000

Optimal solutions

By using the PAYG equilibrium equation

$$\pi_t = D_t \delta_t$$

and by optimizing the loss function according to D , we obtain

$$\delta_t^* = \frac{(1 - \rho) \frac{1}{\bar{\delta}} + \rho \frac{D_t}{\bar{\pi}}}{(1 - \rho) \frac{1}{\bar{\delta}^2} + \rho \frac{D_t^2}{\bar{\pi}^2}}$$

$$\pi_t^* = D_t \delta_t^*$$

$$\bar{\delta} = \delta_0 \frac{(1 - \rho) + \rho \frac{D_0^2}{D_\infty^2}}{(1 - \rho) + \rho \frac{D_0}{D_\infty}}$$

$$\bar{\pi} = \delta_0 \frac{(1 - \rho) D_\infty^2 + \rho D_0^2}{(1 - \rho) D_\infty + \rho D_0}$$

Optimal solutions

Extreme DB and DC schemes

$$\delta_t^* = \frac{(1 - \rho) \frac{1}{\bar{\delta}} + \rho \frac{D_t}{\bar{\pi}}}{(1 - \rho) \frac{1}{\bar{\delta}^2} + \rho \frac{D_t^2}{\bar{\pi}^2}}$$

$$\pi_t^* = D_t \delta_t^*$$

$$\text{DB : } \rho = 0 \quad \begin{cases} \delta_t^* = \bar{\delta} \\ \pi_t^* = D_t \bar{\delta} \end{cases} \quad \text{DC : } \rho = 1 \quad \begin{cases} \delta_t^* = \frac{\bar{\pi}}{D_t} \\ \pi_t^* = \bar{\pi} \end{cases}$$

Risk sharing within our adapted NDC system

Risk sharing determines the contribution rate π and the benefit ratio δ .

We deduce the contribution adjustment rate λ and the sustainability rate β

$$\pi_t^* = \tilde{\pi} (1 + \lambda_t^*)$$

$$\delta_t^* = \frac{1}{\bar{S}_t} \int_{x_r}^{\omega} P_{t-(x-x_r)} \beta_{t-(x-x_r)}^{*x-x_r} l_{x,t} dx$$

Numerical application of the risk sharing

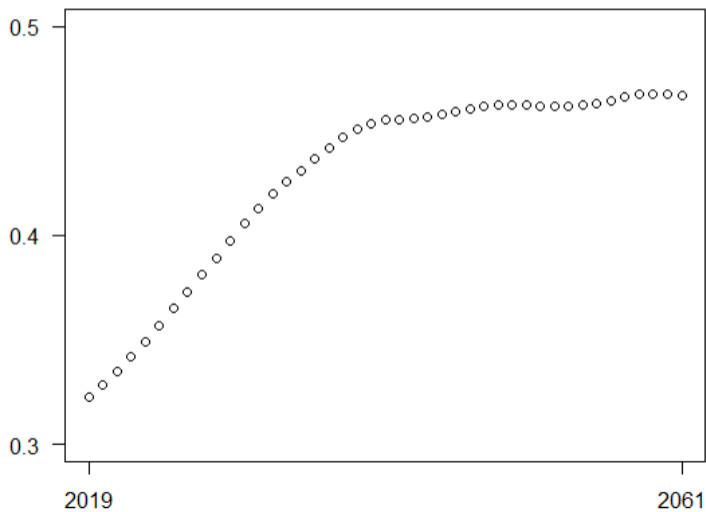
Dependence ratio

Initialization

Risk sharing

Dependence ratio

On data of the Belgian population



Dependence ratio process

The dependence ratio is a mean reverting process and follows a lognormal distribution :
the [Black-Karasinski model](#)

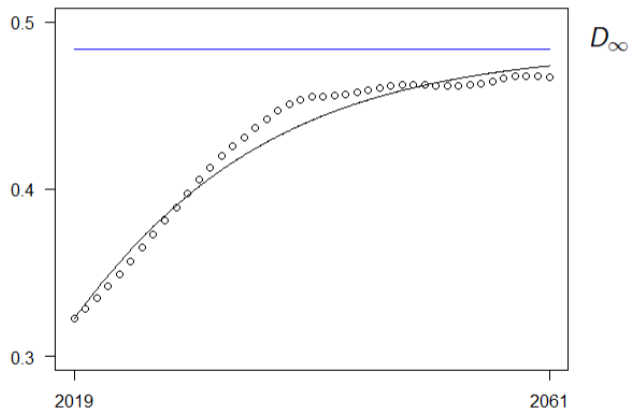
$$d \ln D(t) = \alpha (\ln D_\infty - \ln D(t)) dt + \sigma dW(t)$$

where $\alpha > 0$, $\sigma > 0$ and W_t is a standard Brownian motion.

Calibration using least square regression provides the parameters

$$\alpha = 0.059, \quad D_\infty = 0.47 \quad \text{and} \quad \sigma = 0.0046$$

Dependence ratio process



Initial conditions

The initialisation of our model in $t_0 = 2019$:

- Dependence ratio : $D_0 = 32\%$
- Contribution rate : $\pi_0 = 16\%$
- Benefit ratio : $\delta_0 = 50\%$

Risk sharing

Between workers and retirees

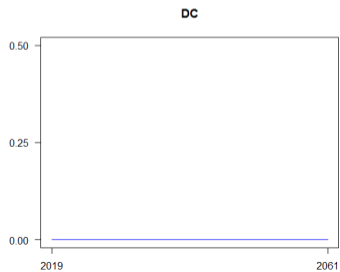
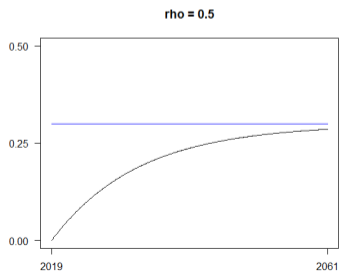
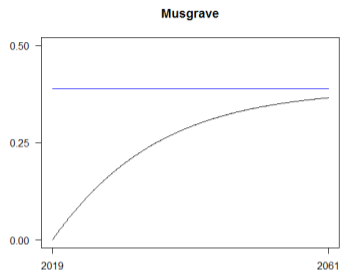
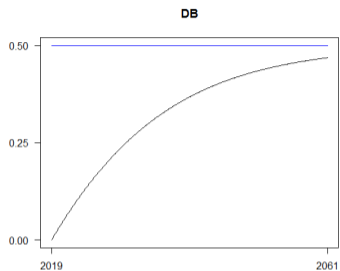
We simulate four scenarios :

DB, DC, Musgrave and $\rho = 0.5$.

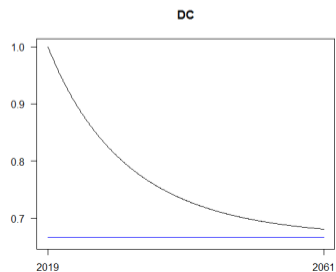
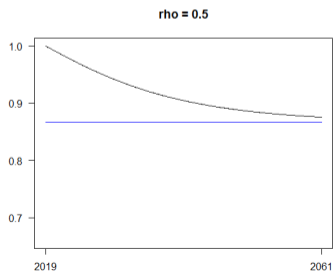
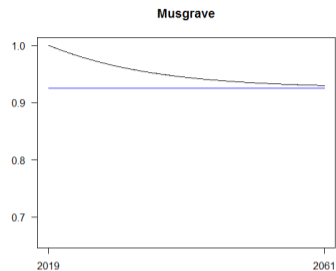
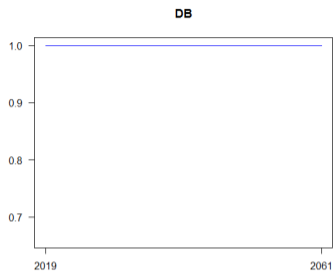
Remember, the loss function to minimize is

$$(1 - \rho) \left(\frac{\delta_t}{\bar{\delta}} - 1 \right)^2 + \rho \left(\frac{\pi_t}{\bar{\pi}} - 1 \right)^2$$

Increase of the contributions for the workers – λ



Decrease of the benefits for the retirees – β



The Musgrave rule and the fifty-fifty risk sharing are two possible hybrid systems sharing the demographic risk.

Which one is the best risk sharing?

Comments on risk sharing

Notion of equity

Musgrave rule

The same relative change for the pension benefits and for the salaries net of contribution

$$\frac{P_{t+1}}{P_t} = \frac{S_{t+1} (1 - \pi_{t+1})}{S_t (1 - \pi_t)}$$

Risk sharing ($\rho = 0.5$)

Fifty-fifty sharing of the demographic risk between the relative increase of the contributions and the relative decrease of the benefits.

Comments on risk sharing

Notion of equity

The Musgrave rule and the risk sharing with $\rho = 0.5$ correspond to two different concepts of equity between the workers and the retirees.

Choosing the optimal sharing corresponds to choose the desired level of equity for the system.

Conclusions

Ageing induces an ineluctable and significant increase of the dependence ratio in the coming decades and either a dramatic increase of the contributions or decrease of the benefits.

In the aim to maintain a financial sustainable and social adequate system, we propose to adapt the traditional NDC system to a hybrid system with a **variable contribution rate**.

This hybrid system **shares the demographic risk** between the workers through their contributions and the retirees through their benefits.

Future research

- Consider **another adjustments variable** : the notional rate. It will impact the system on its longitudinal projection.
- Study of the **socio-economic heterogeneity** of the life expectancy and study of the according adaptation of the legal retirement age.

A new Notional Defined Contribution system
based on risk sharing

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