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Stochastic Modeling of Mortality and Longevity

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Roadmap

Modeled Sources of Volatility to Mortality / Longevity

- Poisson Risk
- Trend Risk
- Basis Risk
- Extreme Long-Term Events
- Catastrophic Short-Term Events
- Selection / Anti-selection

Sample Results





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

1. Poisson Risk

- Consider the **Law of Large Numbers**:
 - *as a sample size grows, its mean will get closer and closer to the average of the whole population*
- However, we may need to model risk associated with small sample sizes

– Poisson Method Applied to Survival Model:

- *In each scenario, for each life, select a random number between 0 and 1, and compare that value to the cumulative survival rates (${}_t p_x$) and model death as occurring at the first time t at which the random number $> {}_t p_x$*



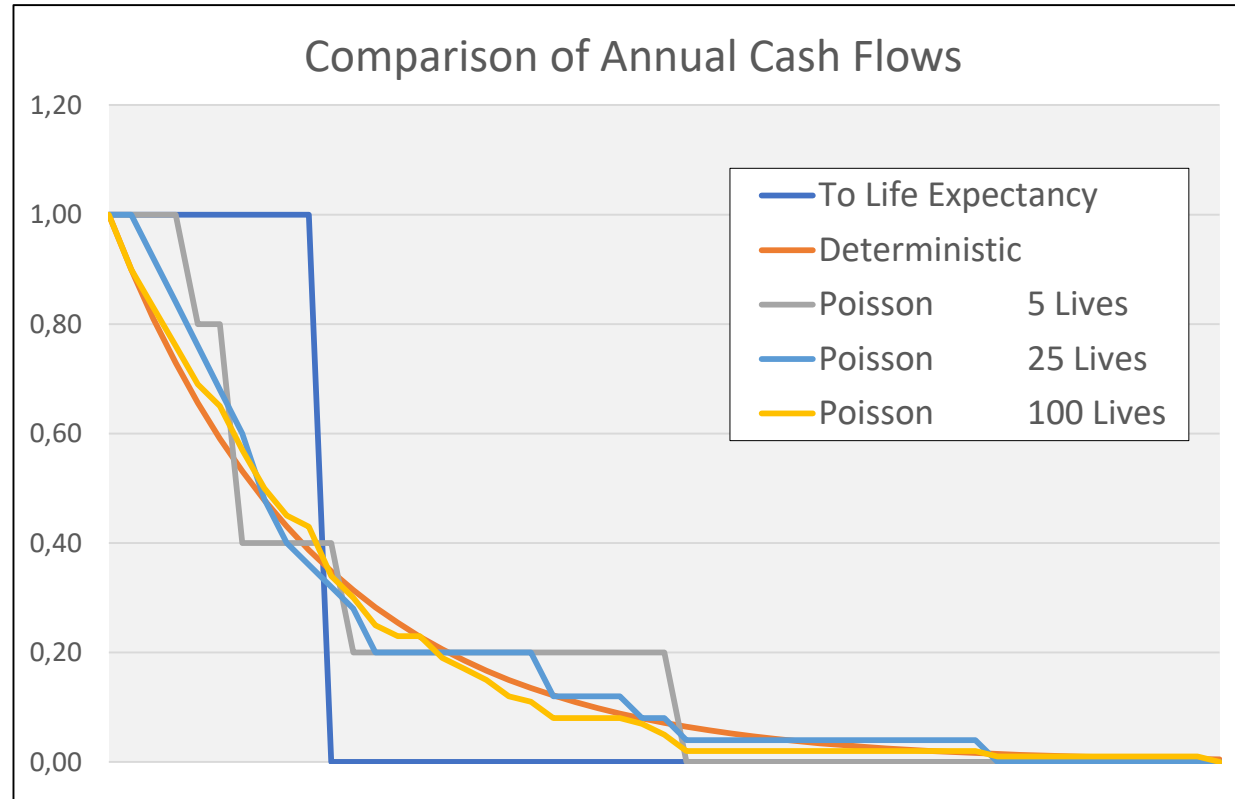


Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

- **Poisson Risk** – Simple Example: \$1 per year payable / Annual Mortality is a constant 10%

Compare Modeling to Life Expectancy to Deterministic and to Poisson





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

- **Poisson Risk** – Simple Example: \$1 per year payable / Annual Mortality is a constant 10%

Compare Modeling to Life Expectancy to Deterministic and to Poisson

Net Present Value of Annual Cash Flow Single Random Scenario					
NPV Future Cashflows at	To Life Expectancy	Deterministic (Probabilistic)	Poisson 5 Lives	Poisson 25 Lives	Poisson 100 Lives
0.00%	\$10.00	\$9.50	\$10.60	\$10.44	\$9.69
5.00%	\$8.11	\$7.00	\$7.64	\$7.41	\$7.01
10.00%	\$6.76	\$5.50	\$6.06	\$5.87	\$5.58





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

2. TREND RISK: Mortality Improvement Volatility

Key parameters for stochastic modeling:

- A. Long term mortality improvement trends
- B. Short-Term (annual) mortality improvement volatility
- C. Correlation in mortality improvement trend volatility across Ages and Genders

Source Used for All Examples: US Population (1970-2010) by Attained Age and Gender

In practice may develop assumptions from relevant population experience

or Company-Specific Data, reflecting those three factors.

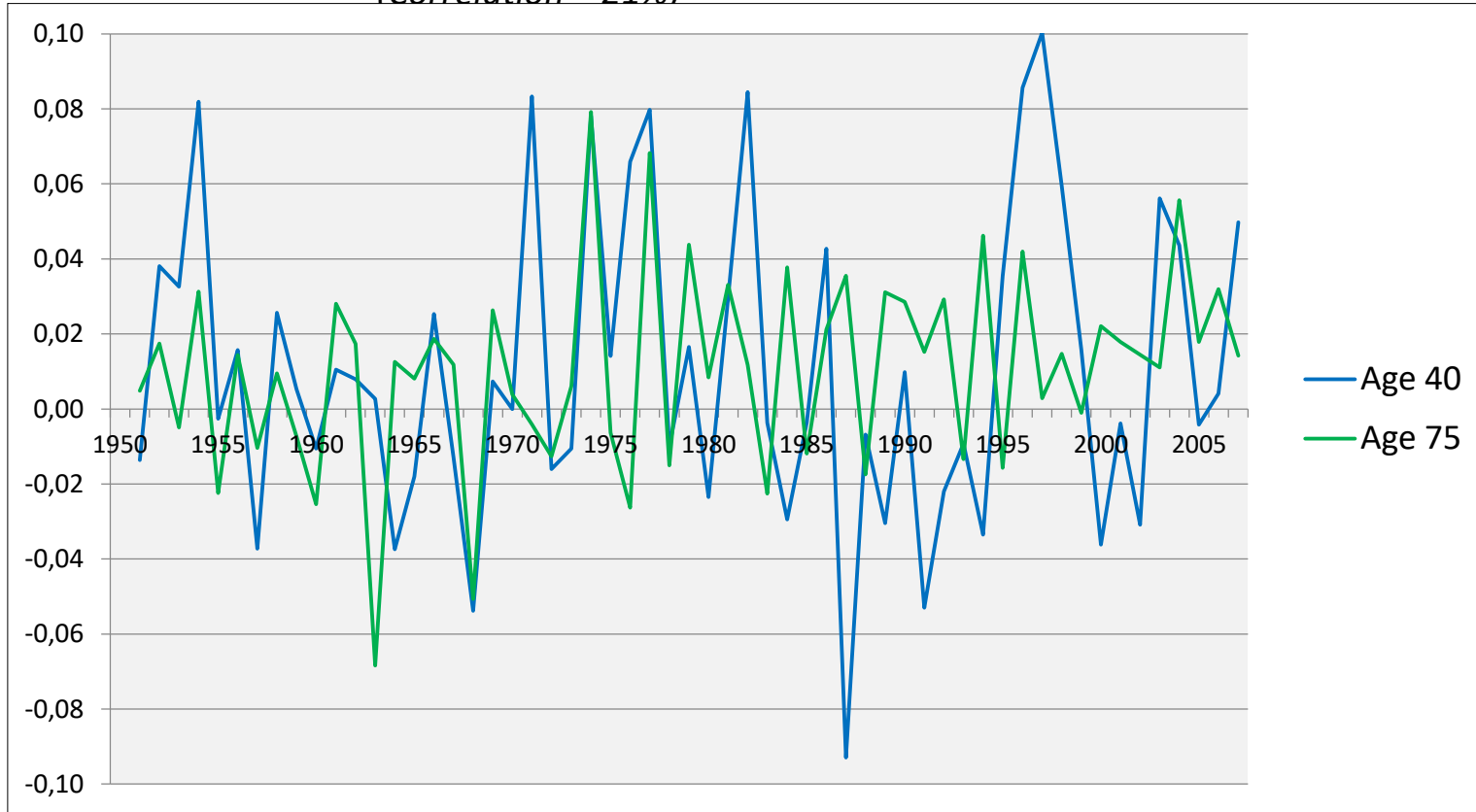




Historical Mortality Improvement

Annual Improvement 1950-2007 Male

(Correlation = 21%)



Std Dev₄₀ = 4.1%

Std Dev₇₅ = 2.6%

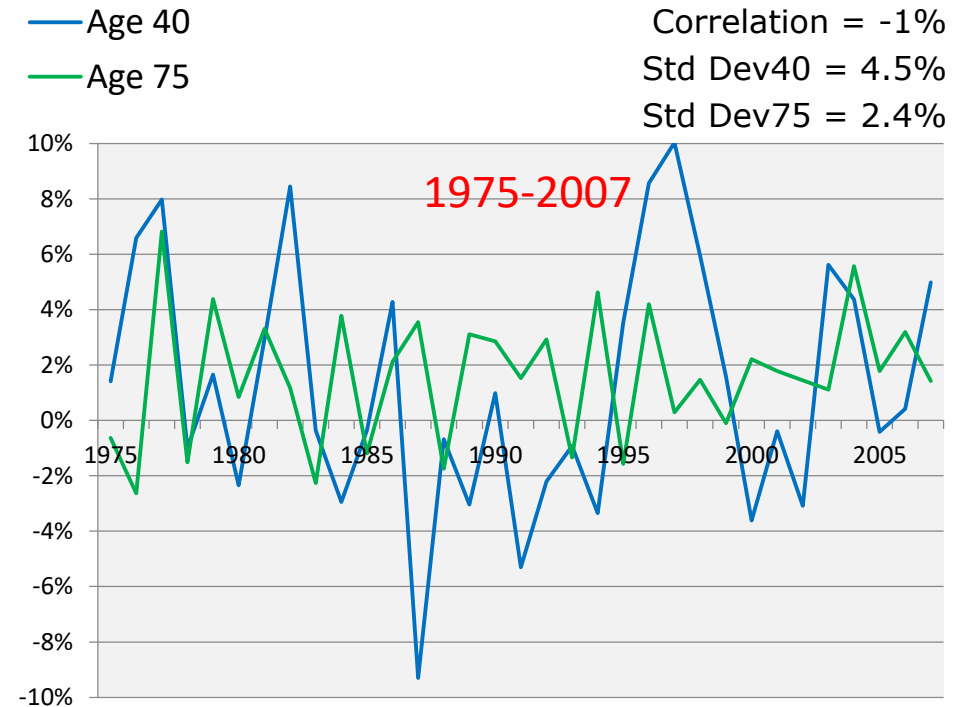
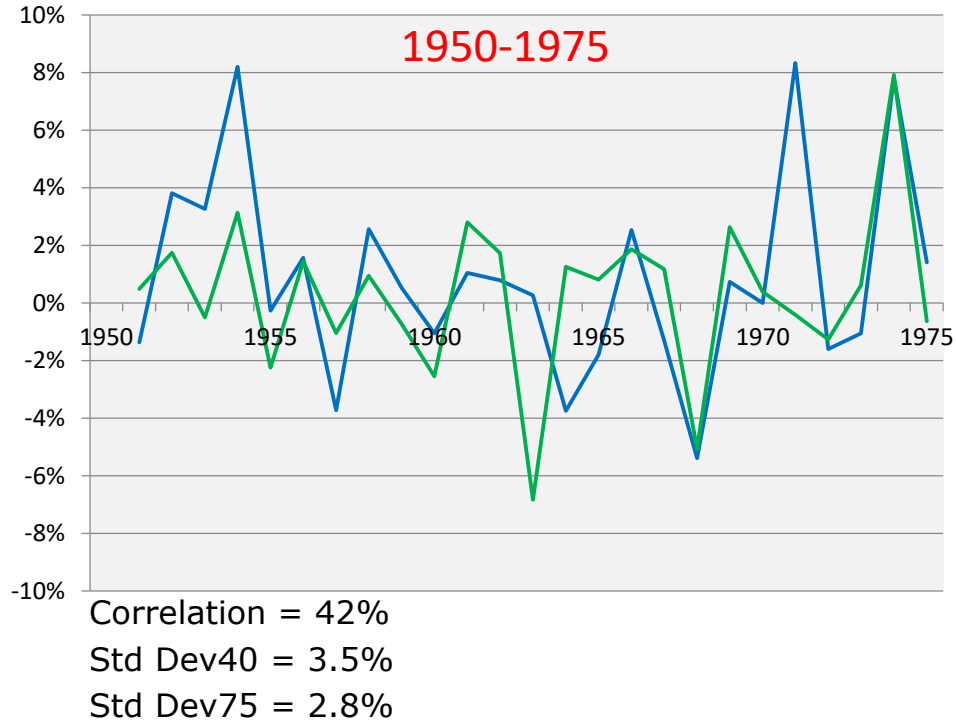
— Age 40
— Age 75





Historical Mortality Improvement

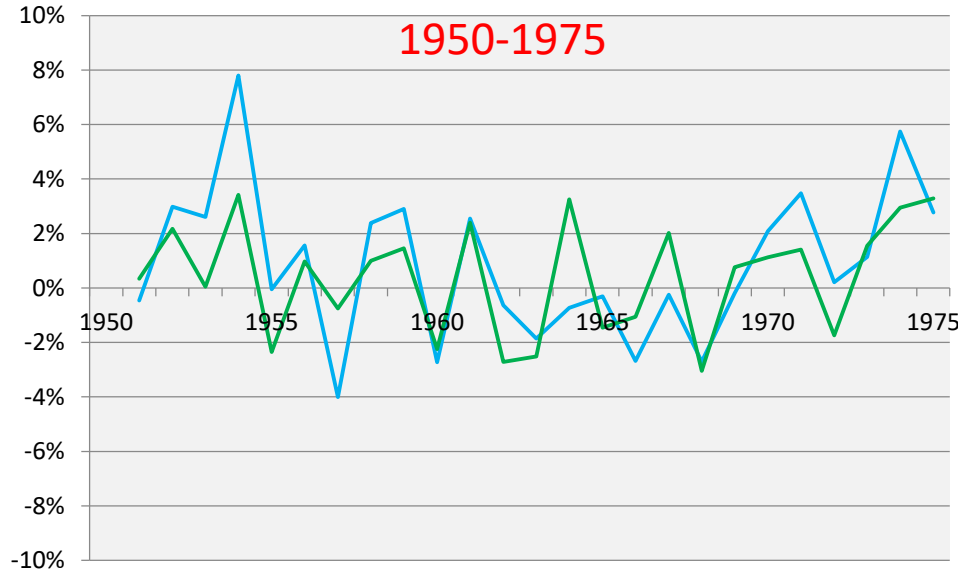
Annual Improvement Male





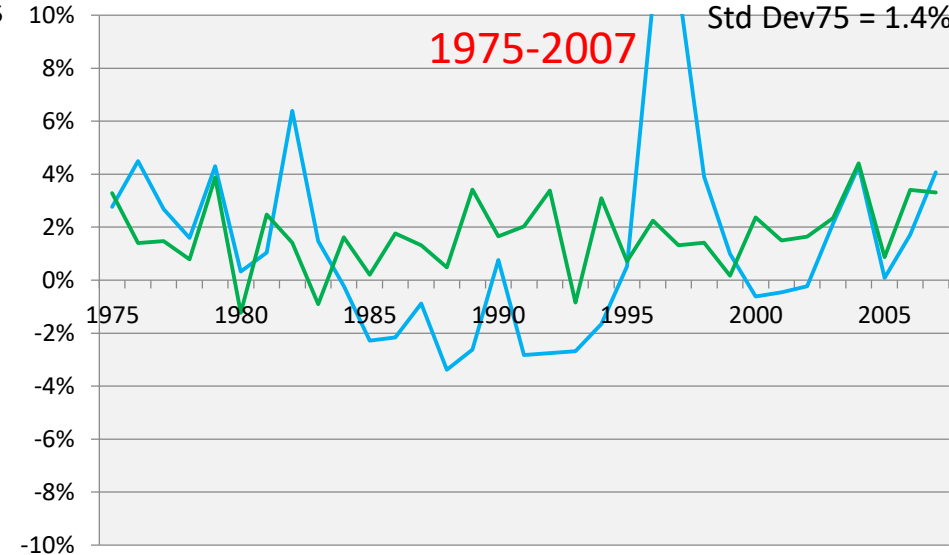
Historical Mortality Improvement

Annual Improvement Male –Age Groups



Correlation = 71%
Std Dev40 = 2.8%
Std Dev75 = 2.1%

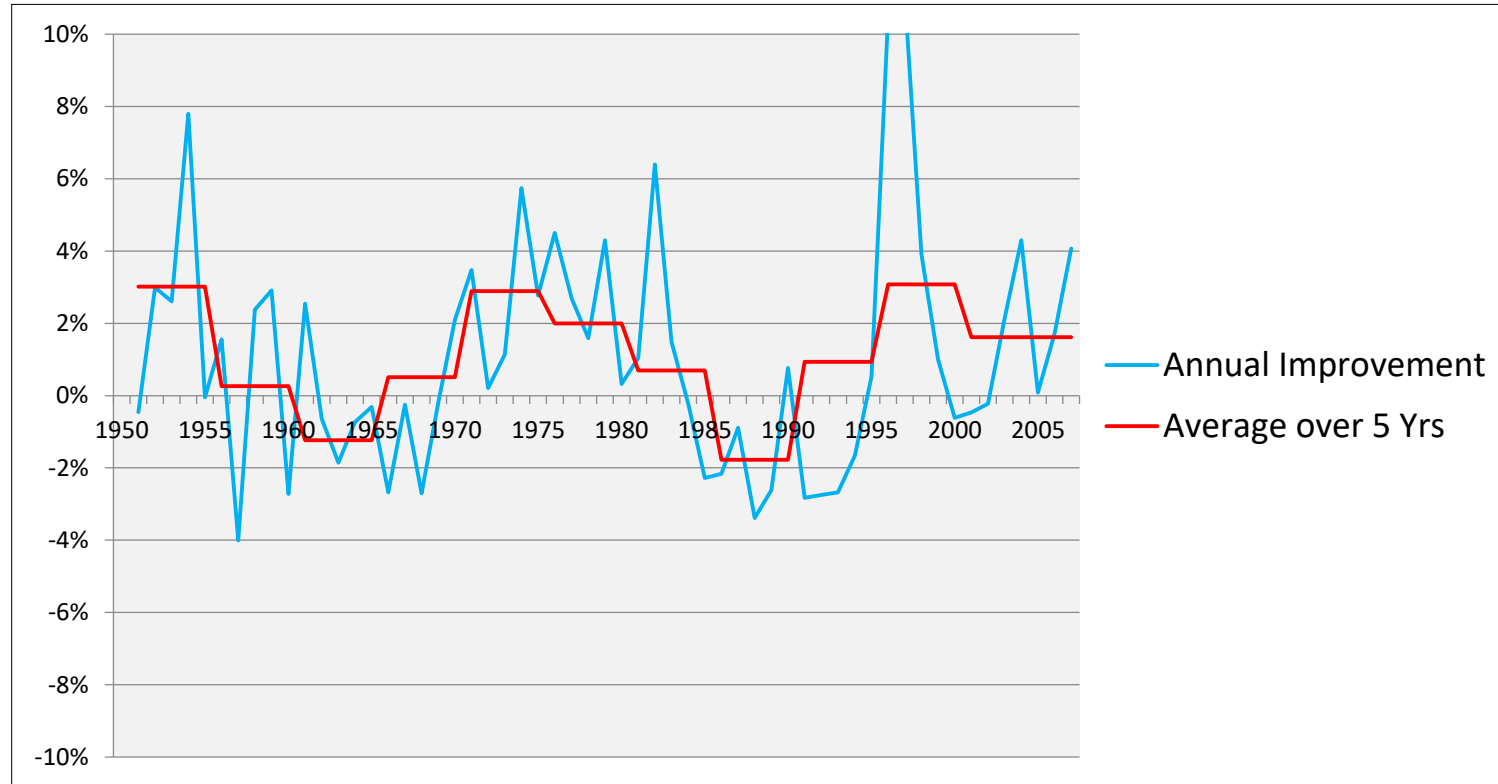
— Ages 38-43
— Ages 73-78
Correlation = 16%
Std Dev40 = 3.5%
Std Dev75 = 1.4%





Historical Mortality Improvement

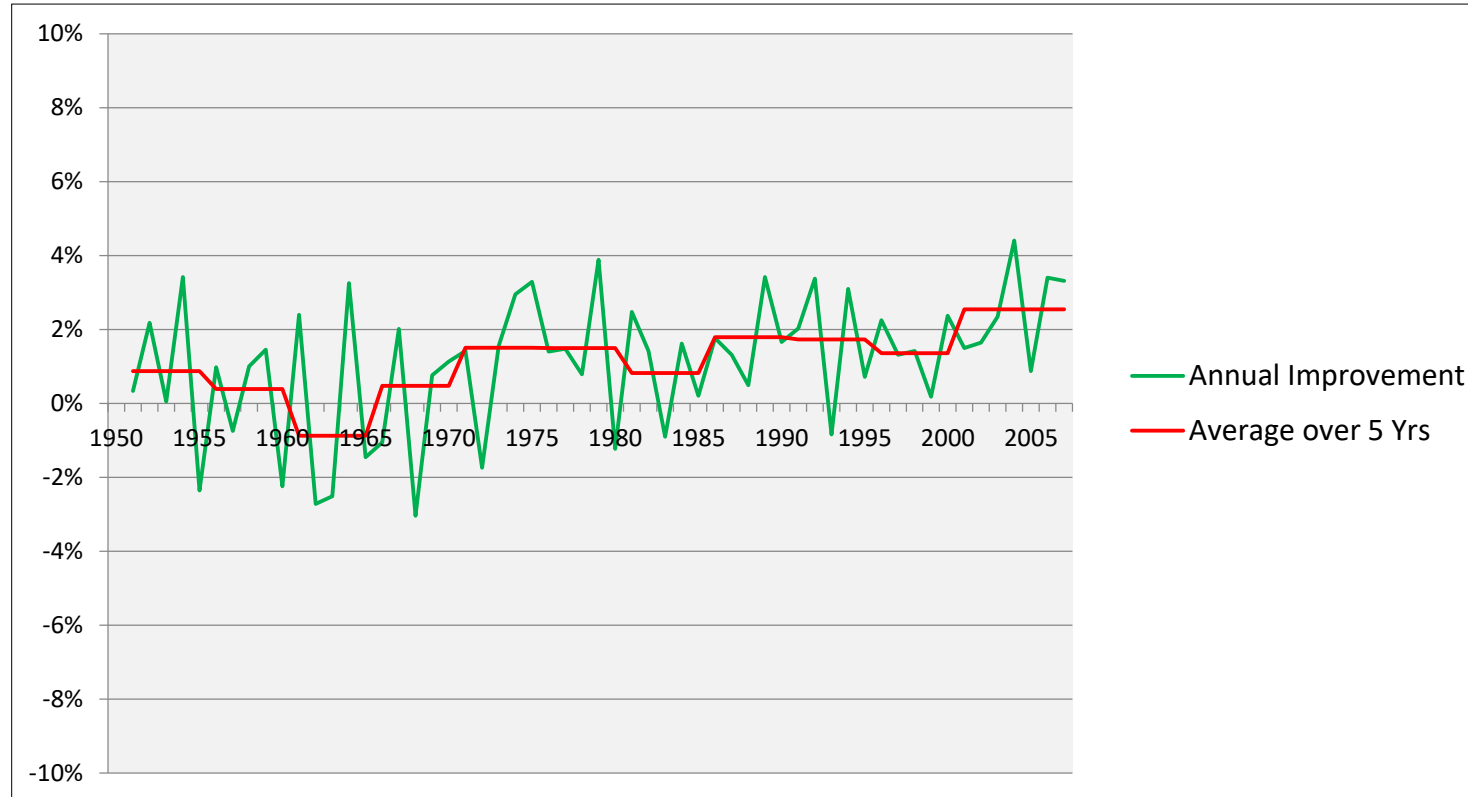
Mortality Improvement 1950-2007 Male Ages 38-43





Historical Mortality Improvement

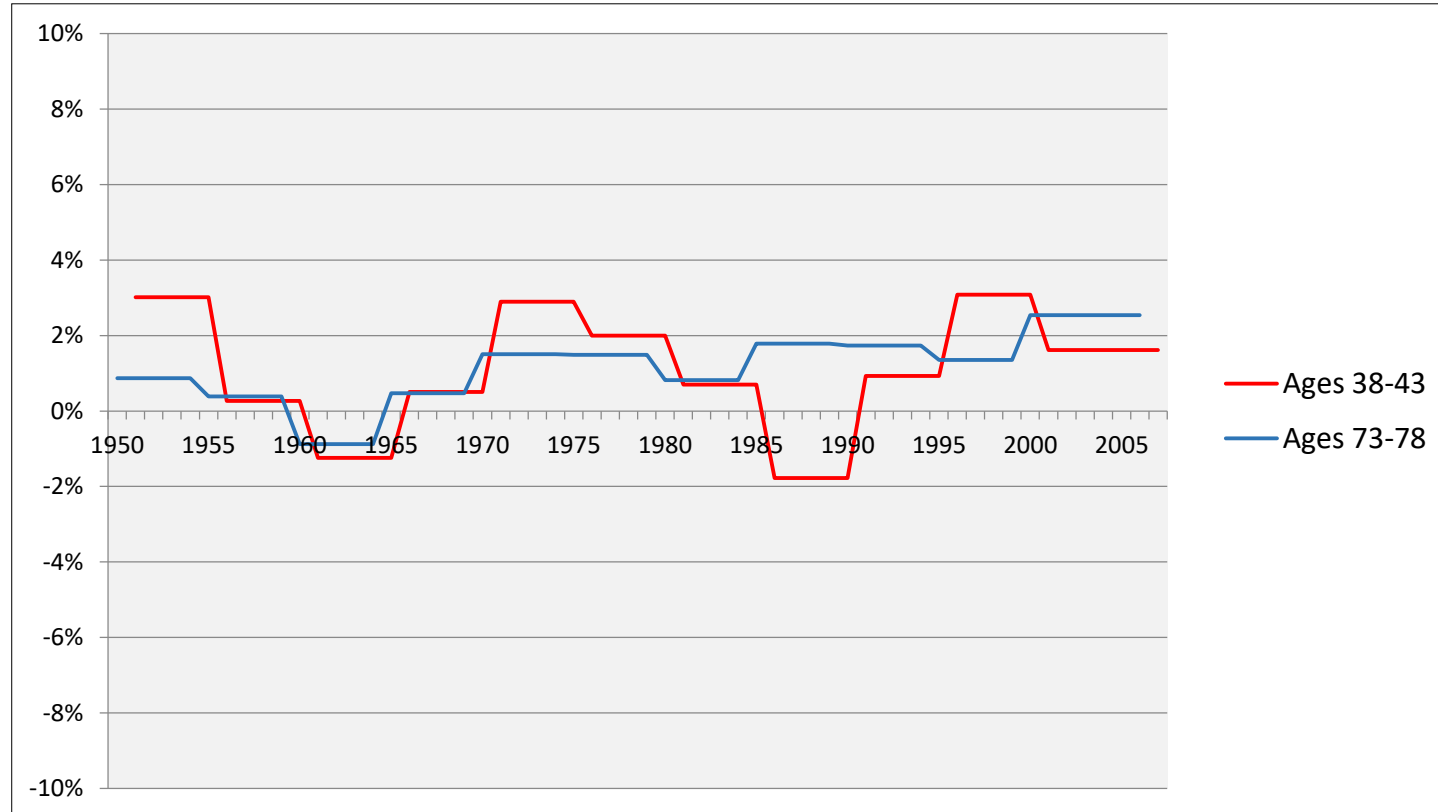
Mortality Improvement 1950-2007 Male Ages 73-78





Historical Mortality Improvement

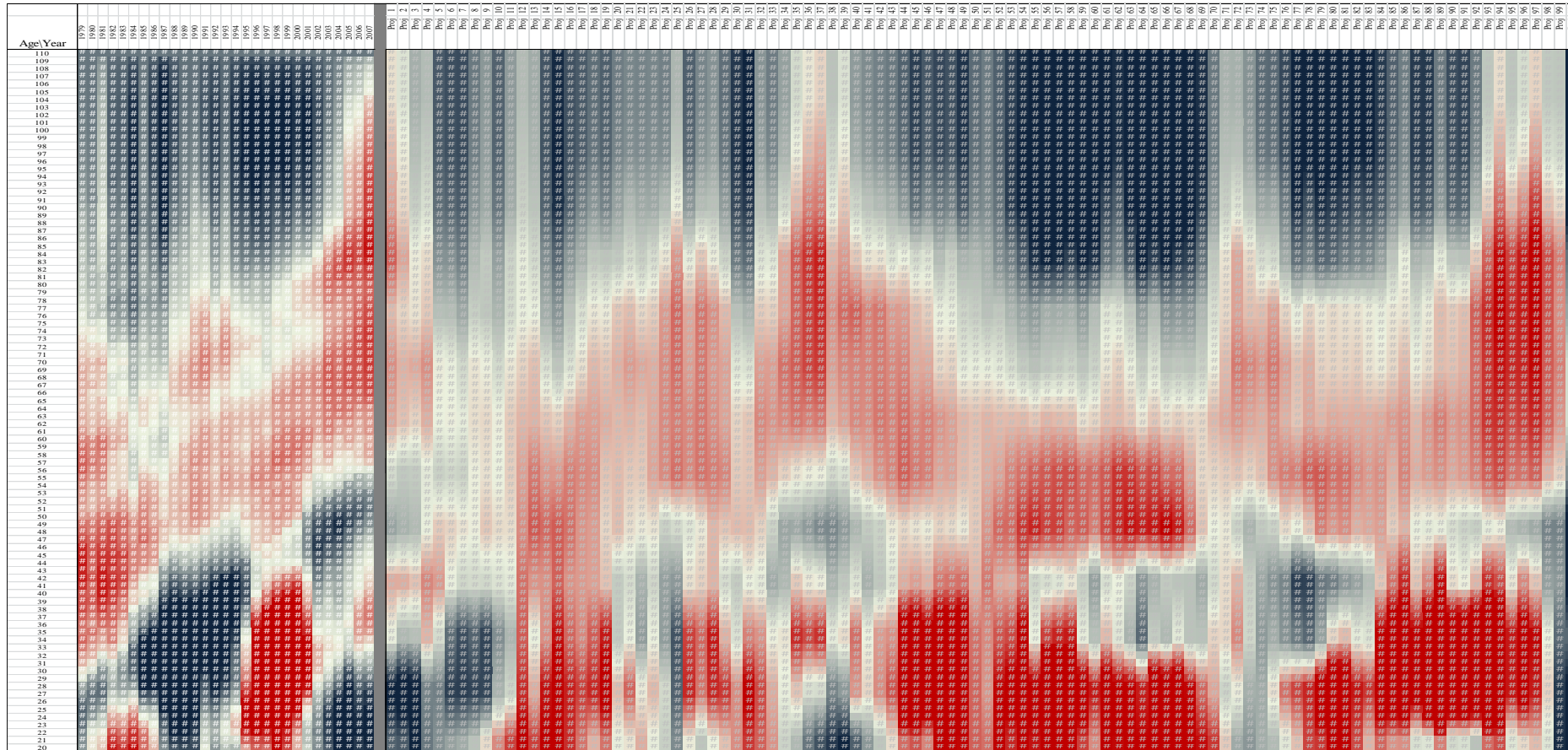
Mortality Improvement 1950-2007 Male – Average Over 5 Years





Stochastic Mortality Improvement

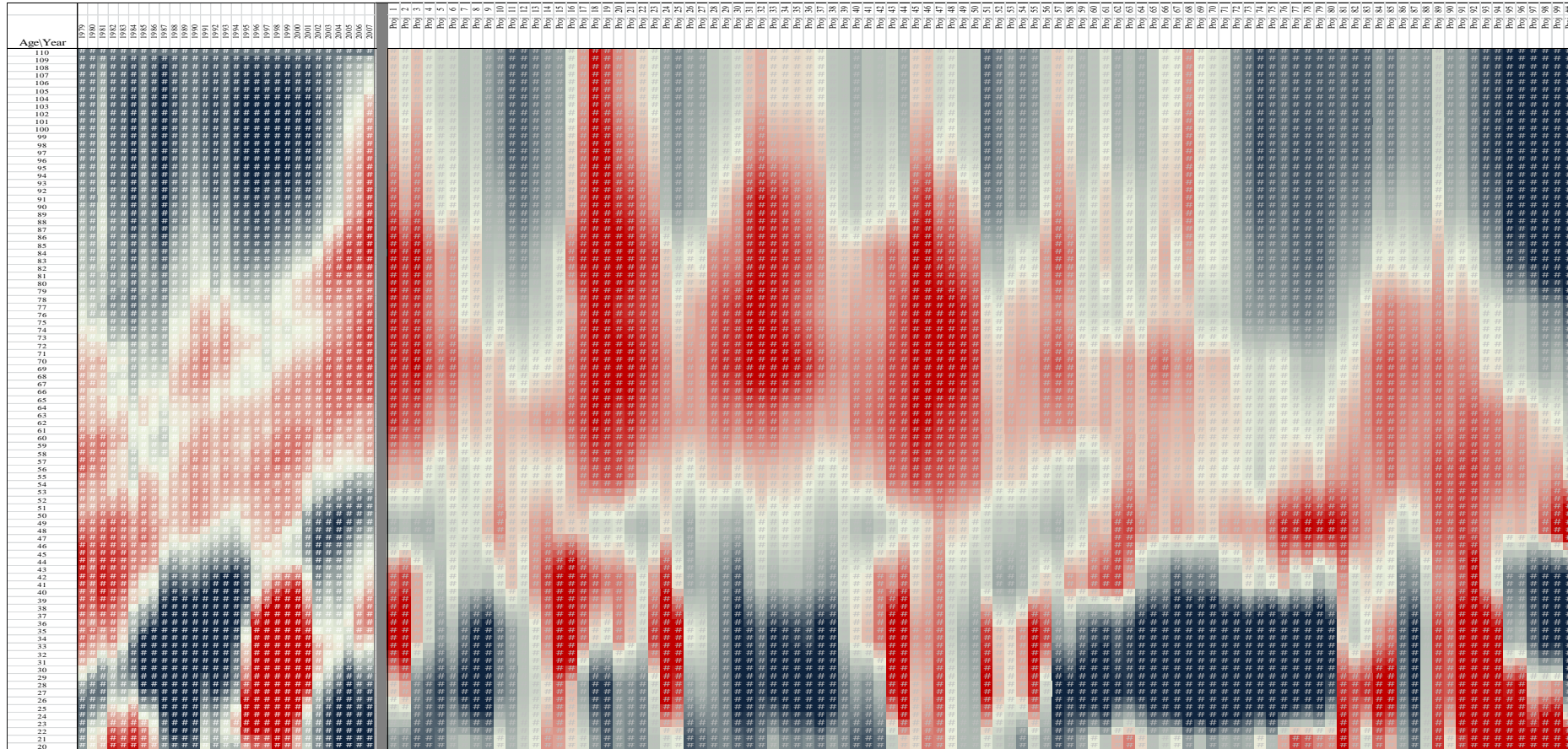
Historic and Scenario 1 Sample Stochastic Projections for 100 years





Stochastic Mortality Improvement

Historic and Scenario 2 Sample Stochastic Projections for 100 years



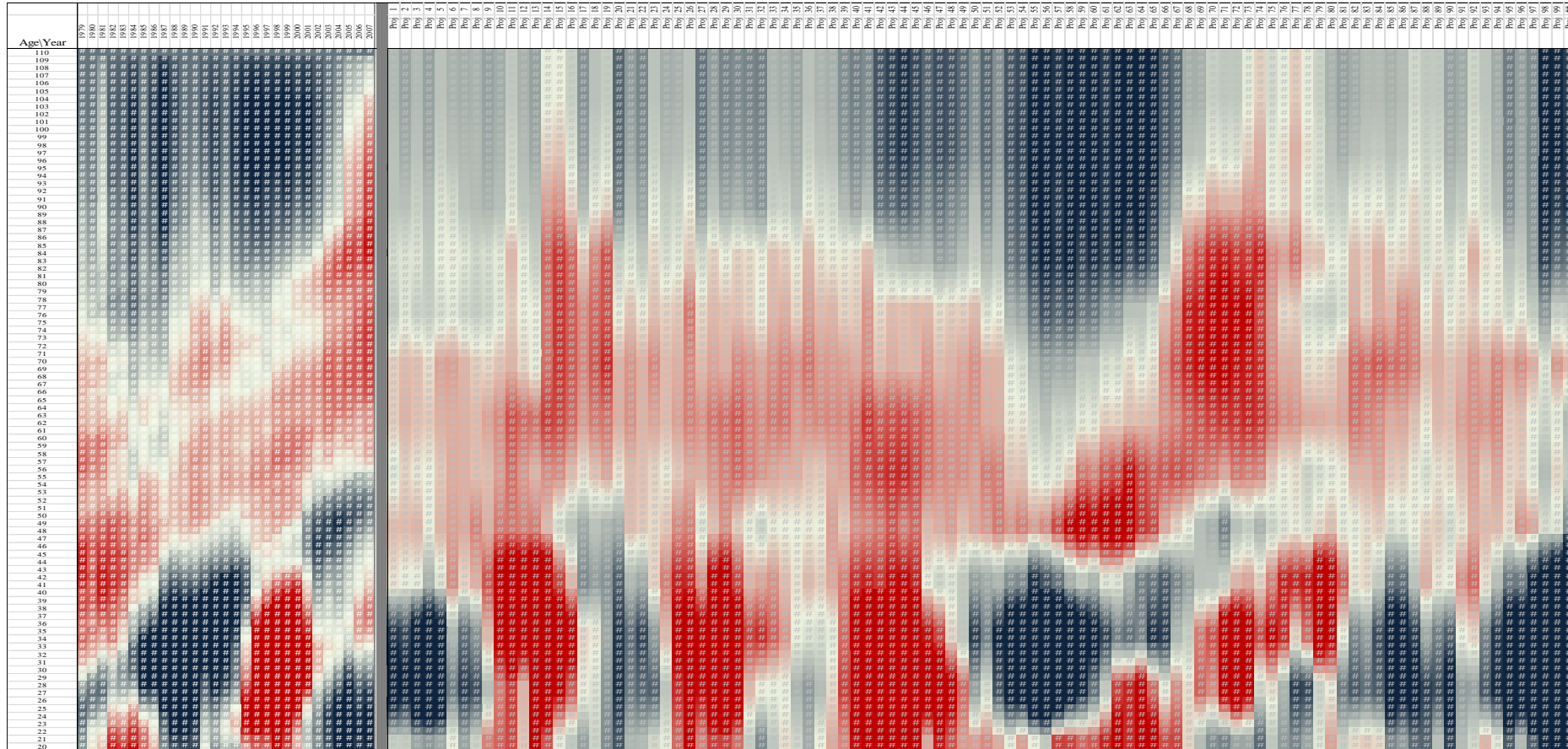
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Stochastic Mortality Improvement

Historic and Scenario 3 Sample Stochastic Projections for 100 years



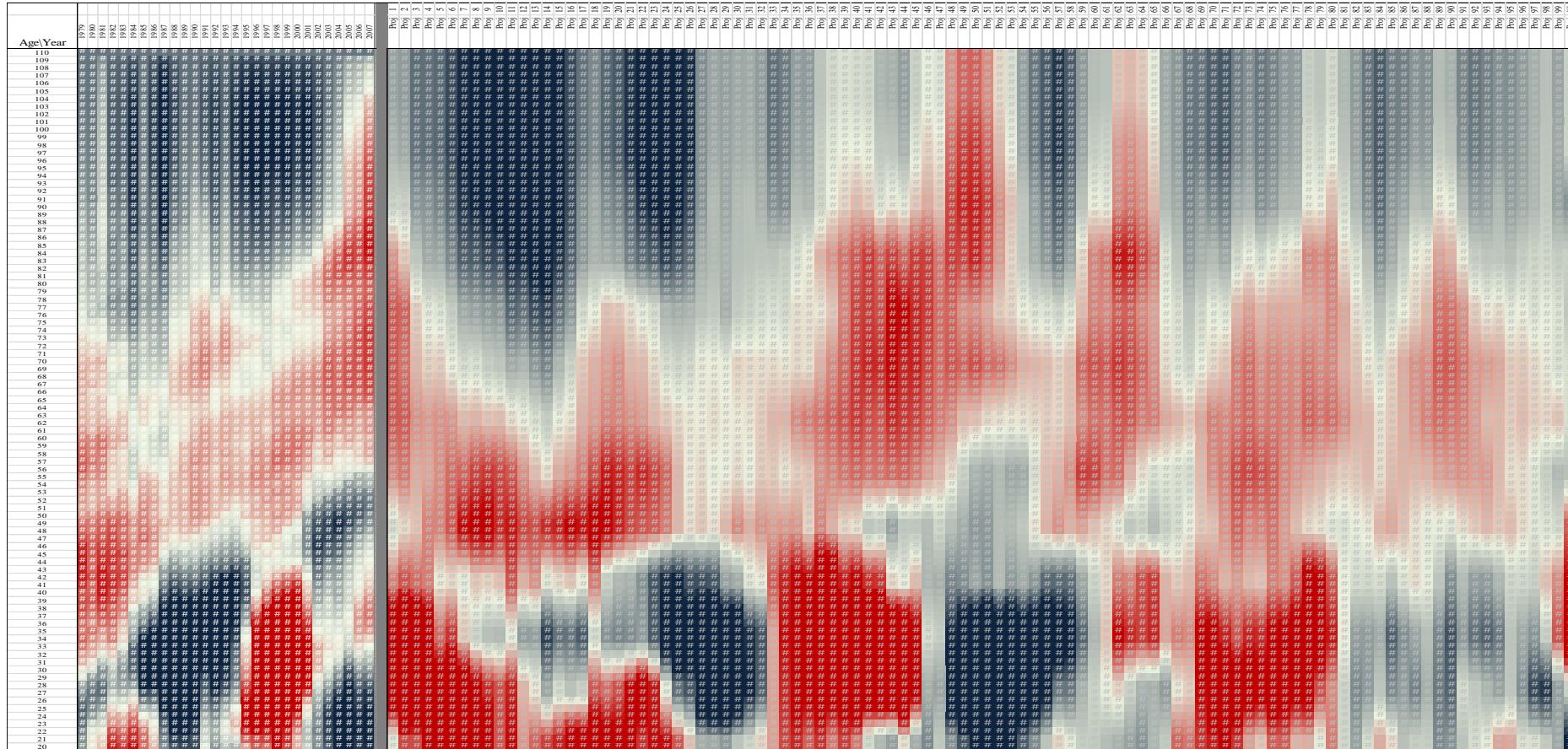
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Stochastic Mortality Improvement

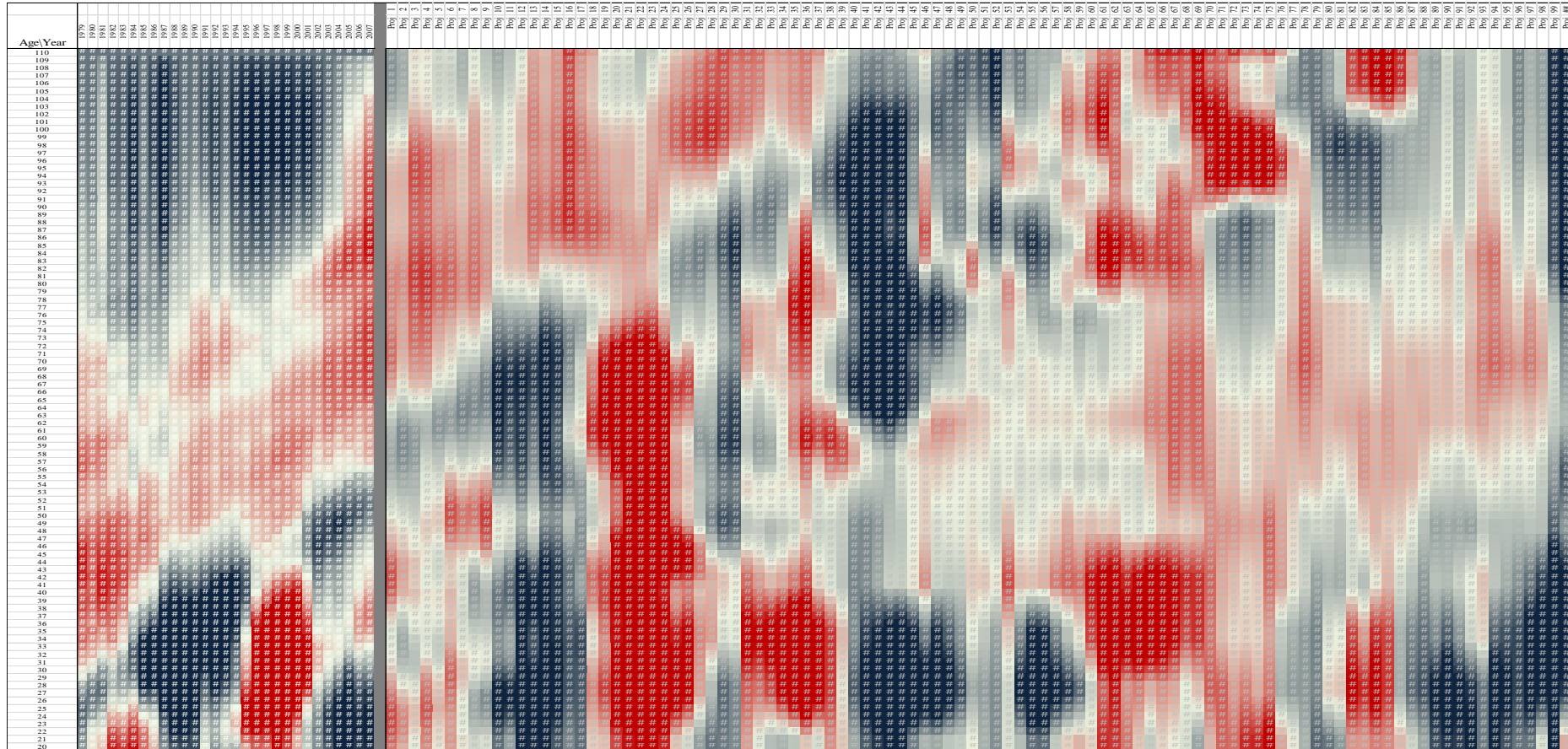
Historic and Scenario 4 Sample Stochastic Projections for 100 years





Stochastic Mortality Improvement - (Cohort Effect)

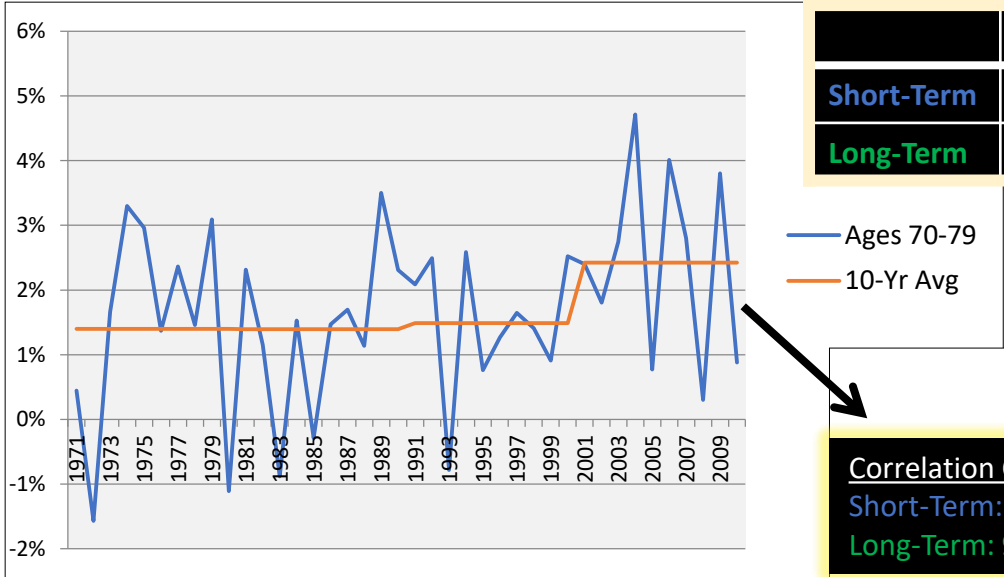
Historic and Alternative Scenario Stochastic Projections for 100 years





Stochastic Mortality Improvement

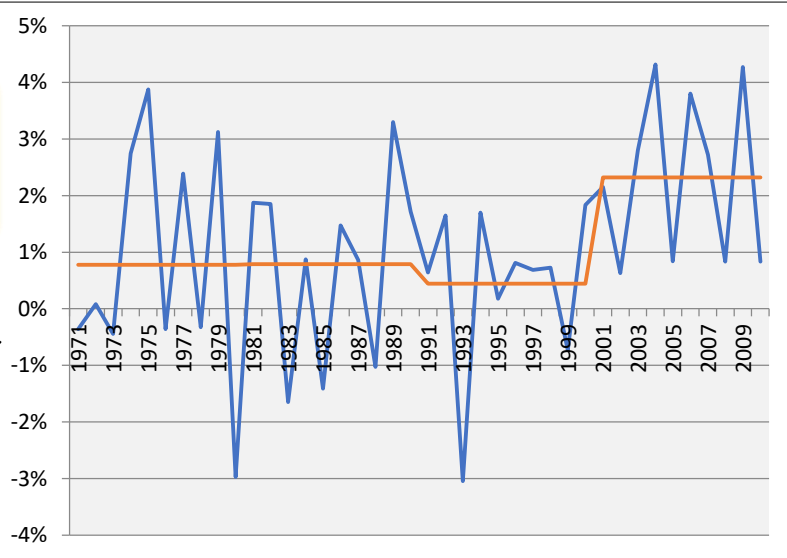
US Mortality Improvement 1970-2010 for Males Ages 70-79 and 80-89



	Std Dev
Short-Term	1.41%
Long-Term	0.50%

Correlation Coefficient
Short-Term: 88.10%
Long-Term: 96.14%

	Std Dev
Short-Term	1.79%
Long-Term	0.84%





Stochastic Mortality Improvement

Comparison of Historical US Population Improvement to Average Results Over 100 Stochastic Projections of Future Mortality Improvement

Short-Term (Annual)	Standard Deviation		Correlation	
	Historic Results	Average Projected	Historic Results	Average Projected
Ages 70-79	1.41%	1.38%	88.10%	88.35%
Ages 80-89	1.79%	1.76%		

Long-Term (10-Yr Periods)	Standard Deviation		Correlation	
	Historic Results	Average Projected	Historic Results	Average Projected
Ages 70-79	0.50%	0.46%	96.14%	94.02%
Ages 80-89	0.84%	0.78%		





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

3. Volatility Around Baseline Expected Mortality Table (*Basis Risk*)

Assumed mortality based on standard industry tables but business placed with any given insurer may reflect different characteristics from the those underlying standard tables.

Annuity example – The risks associated with annuitant lives may vary by occupation, size of policy, or region.

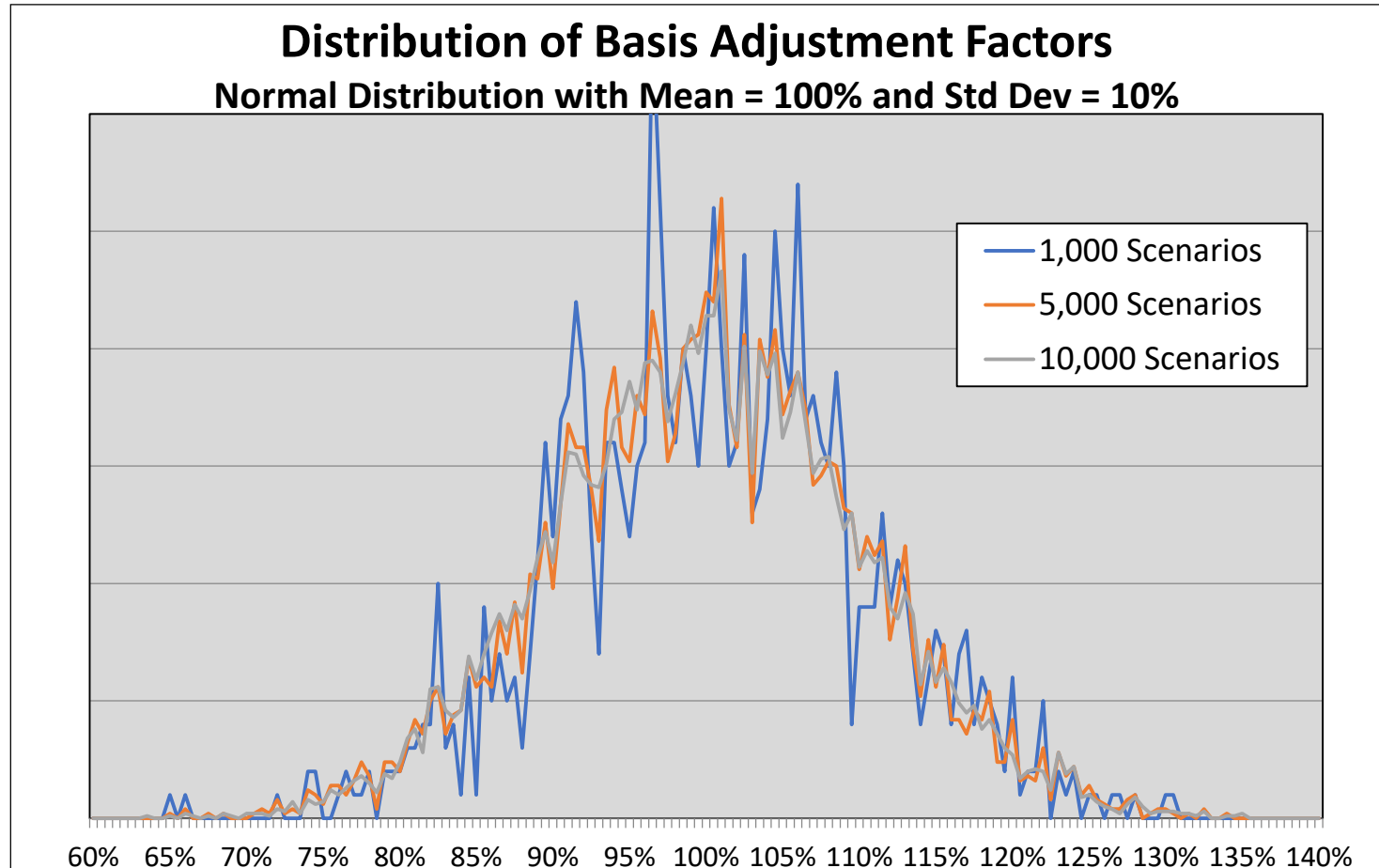
Life insurance example – The underwriting process assigns each life to discrete underwriting classes, each of which may cover a range of expected mortality.





Stochastic Projections of Mortality and Longevity

Volatility Around Baseline Expected Mortality Table





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

4. Mortality Volatility by Cause of Death (Extreme Long Term Events)

Events that cause mortality rates to change faster and more abruptly than anticipated in the other sources.

Examples:

- Effective new treatments for specific diseases
- Evolution of drug-resistant infections

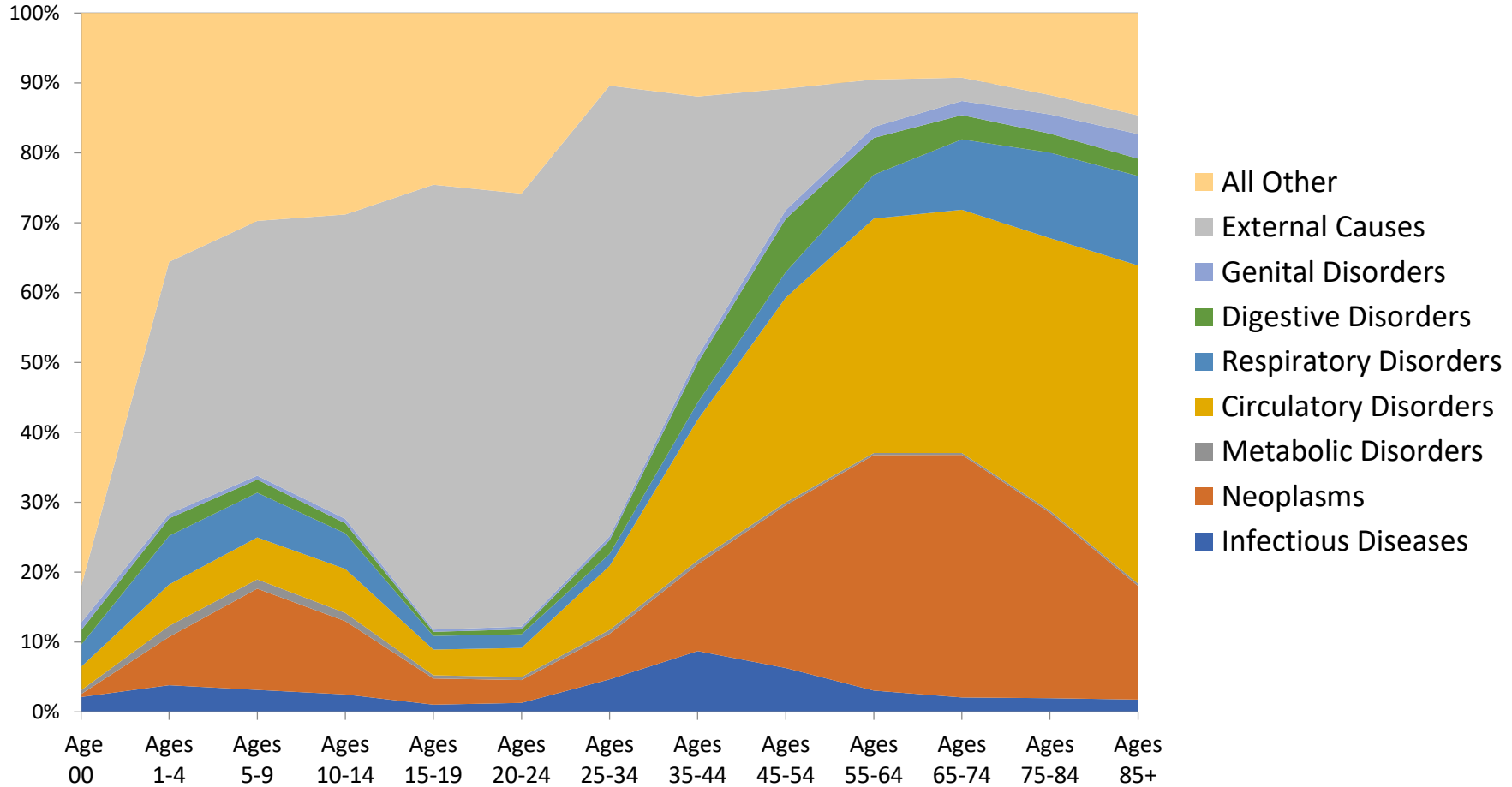
Potential Source: US Population COD Statistics by Attained Age and Gender





Stochastic Projections of Mortality and Longevity

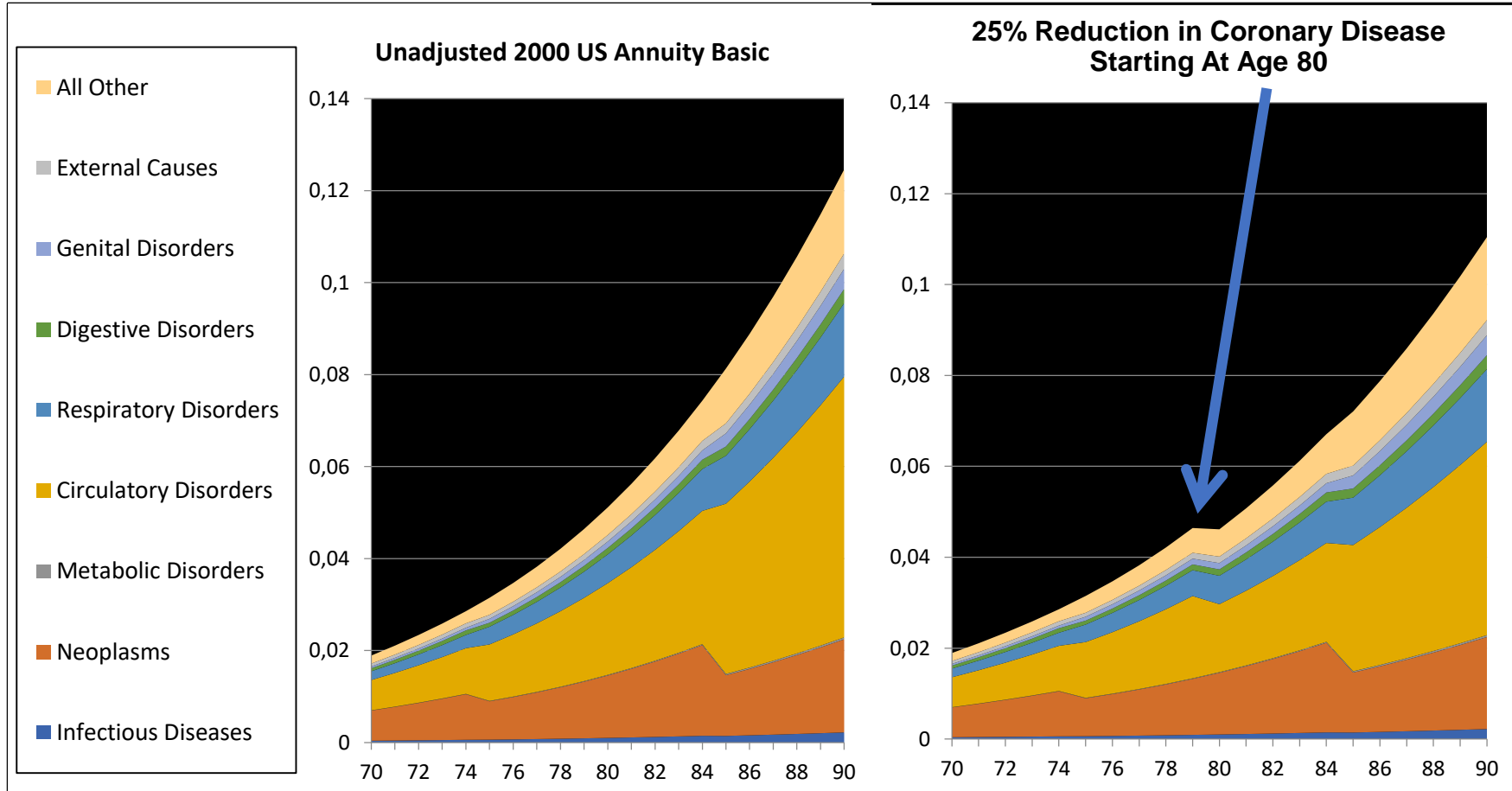
Male Lives (US Population 2000-2010) - Distribution of Deaths by Cause





Stochastic Projections of Mortality and Longevity

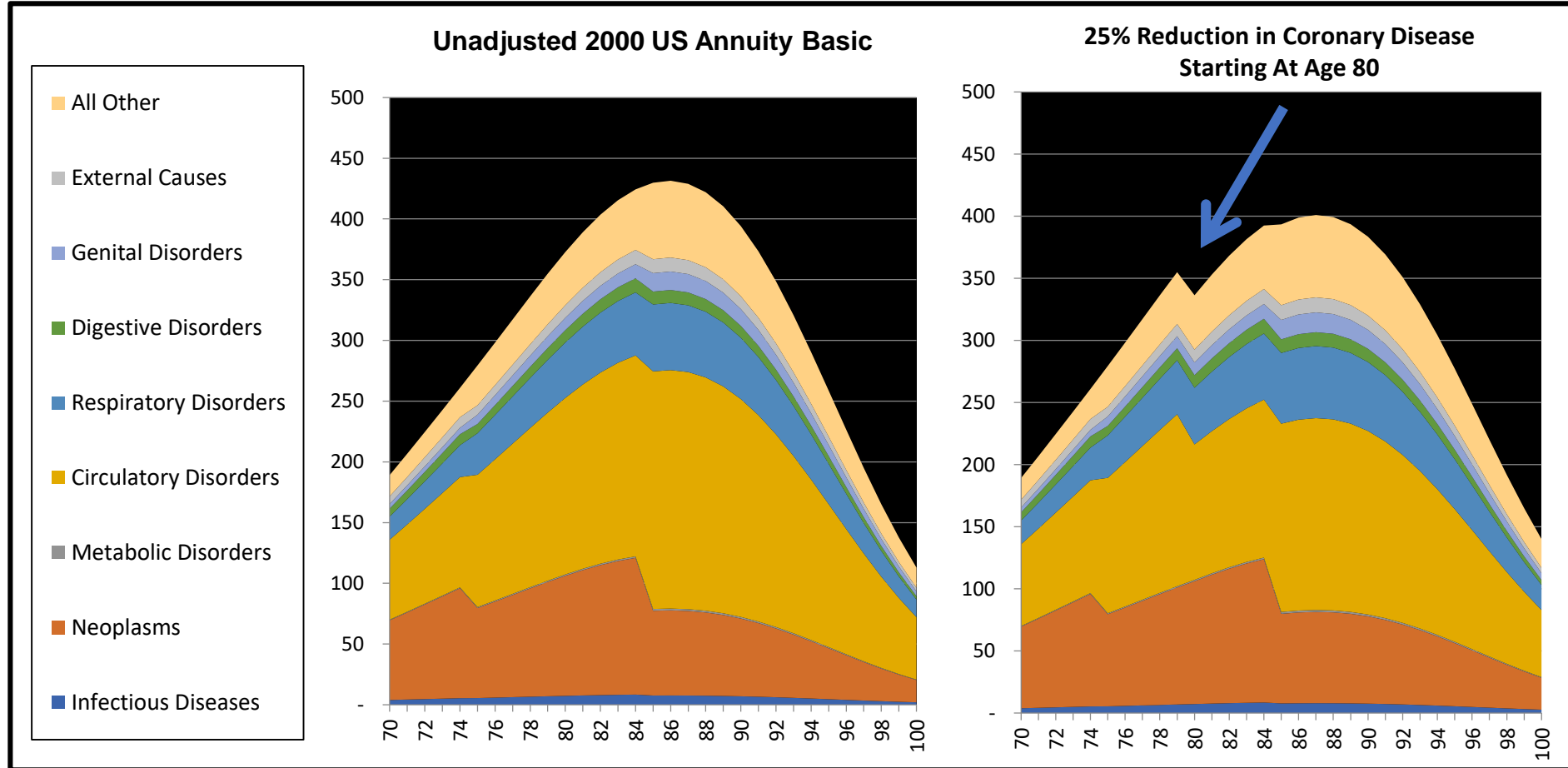
Male Lives (US Population 2000-2010) - Mortality Rate by Cause of Death





Stochastic Projections of Mortality and Longevity

Male Lives (US Population 2000-2010) - Number Deaths Out of 10,000 Lives





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

5. Catastrophes (Catastrophic Short Term Events)

Abrupt temporary deviations in mortality trends

Examples:

- Terrorism
- Flu epidemic
- Natural Disaster





6. Selection / Anti-Selection Effect

Effect of underwriting wears off → Select and Ultimate Mortality Tables
→ preferred or substandard selection risk may wear off, and
→ anti-selection risk may wear off, too

That produces uncertainty:

- a) around the length of the initial selection period,
- b) around years over which preferred / substandard rating takes to wear off
- c) around ultimate level of mortality after the completion of the wearing off.





Stochastic Projections of Mortality and Longevity

MODELED SOURCES OF VOLATILITY

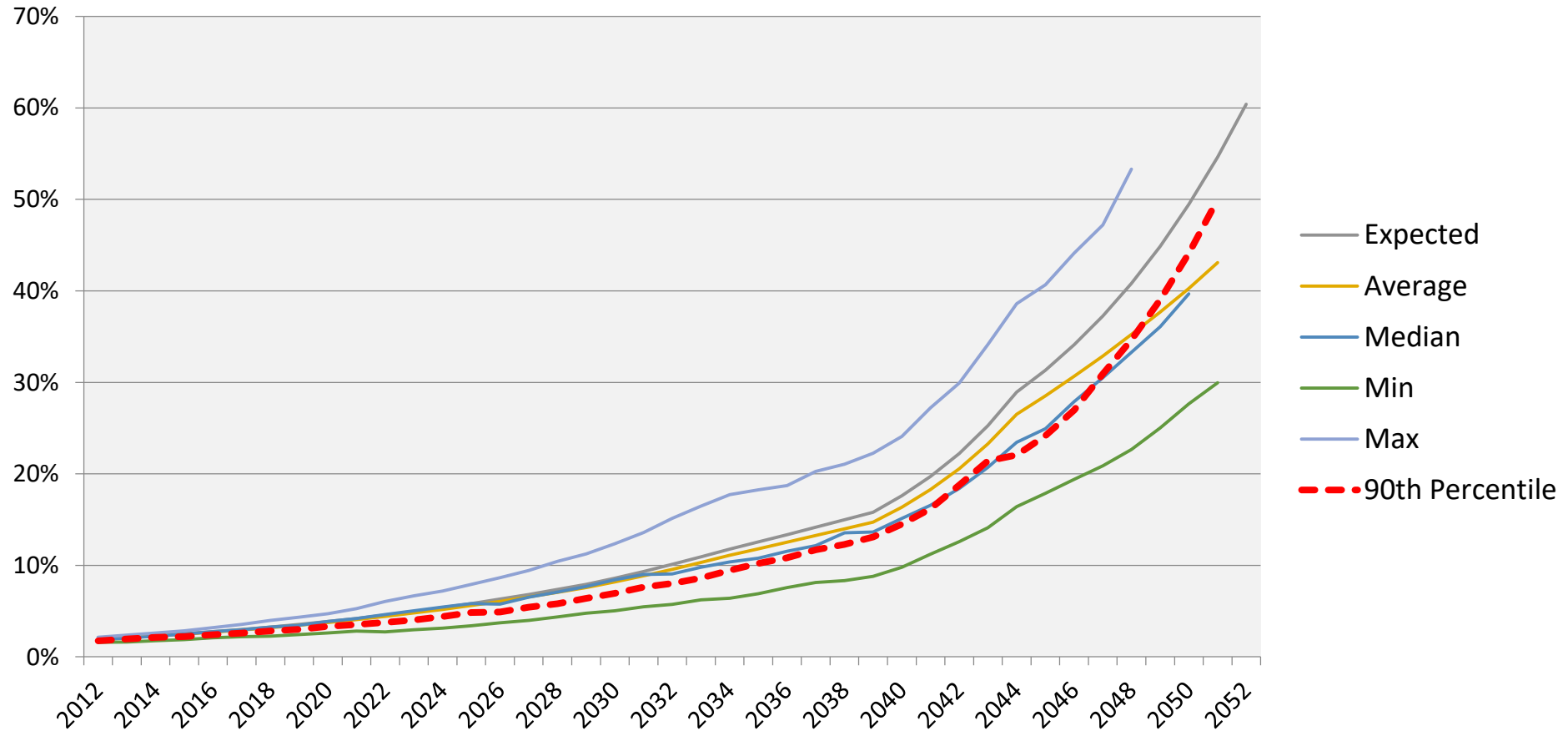
1. Poisson Risk (Individual Date of Death)
2. Mortality Improvement Volatility (Trend Risk)
Adjustments to expected improvement by attained age and calendar year
3. Volatility Around Baseline Expected Mortality Table (Basis Risk)
Scalar applied to all baseline mortality rates before improvement and adjustments
4. Mortality Volatility by Cause of Death (Extreme Long Term Events)
Mortality rates change faster & more abruptly than anticipated in the other sources.
5. Catastrophic Short Term Events
Abrupt temporary deviations in mortality rates
6. Selection / Anti-Selection Risk
How long does it last? What does it grade to?





Stochastic Projections of Mortality and Longevity

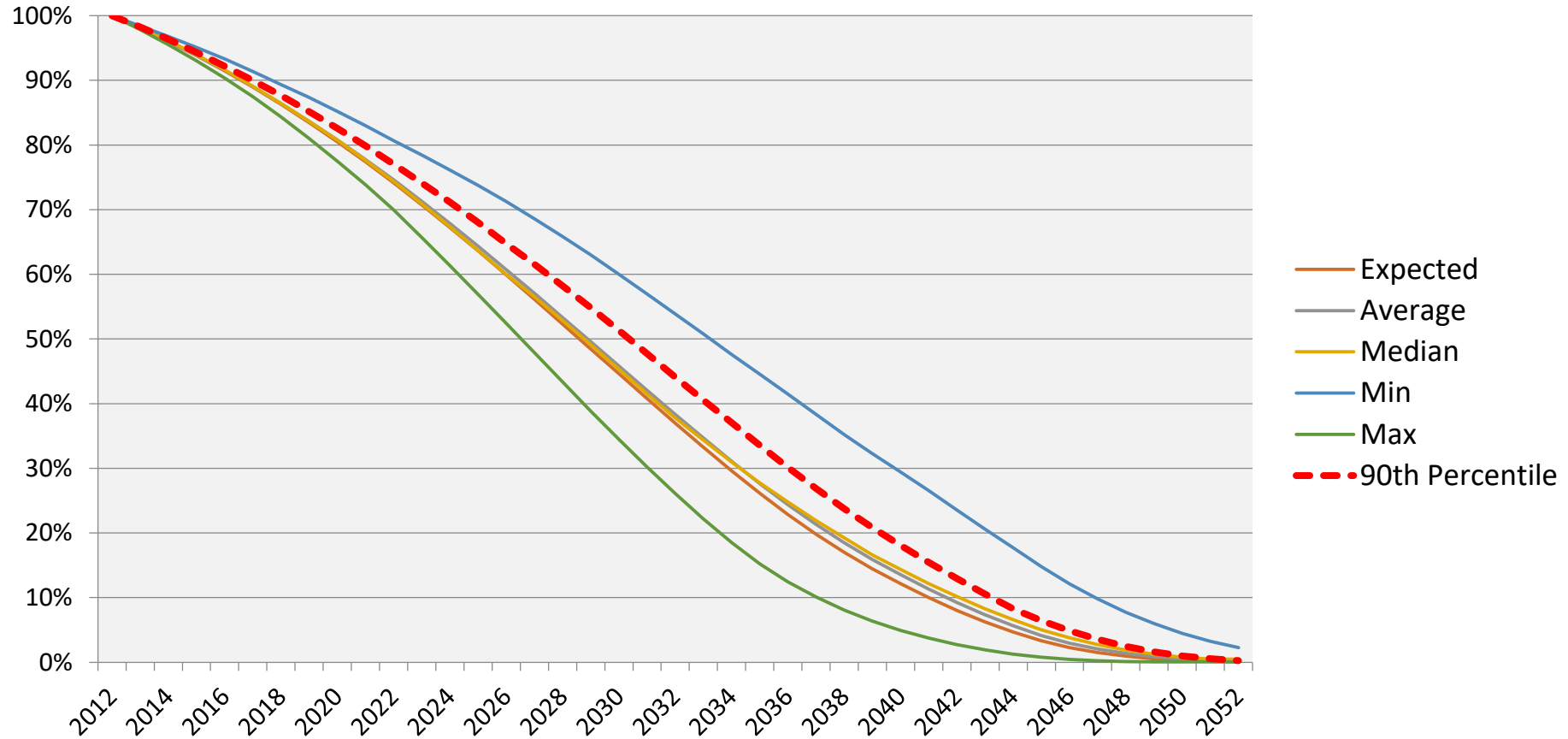
Annual Mortality Rates Ranked by Scenario LE





Stochastic Projections of Mortality and Longevity

Cumulative Survival Ranked by Scenario LE





Thank you very much for your attention!

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