Forward Transition Rates

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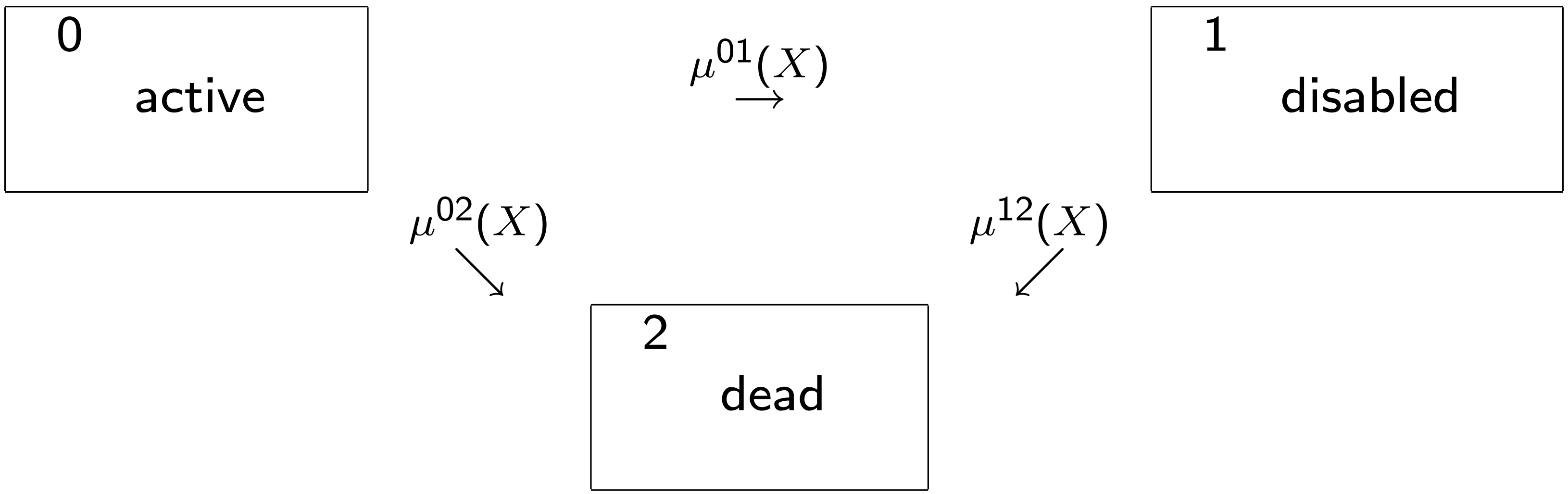

Figure 1:

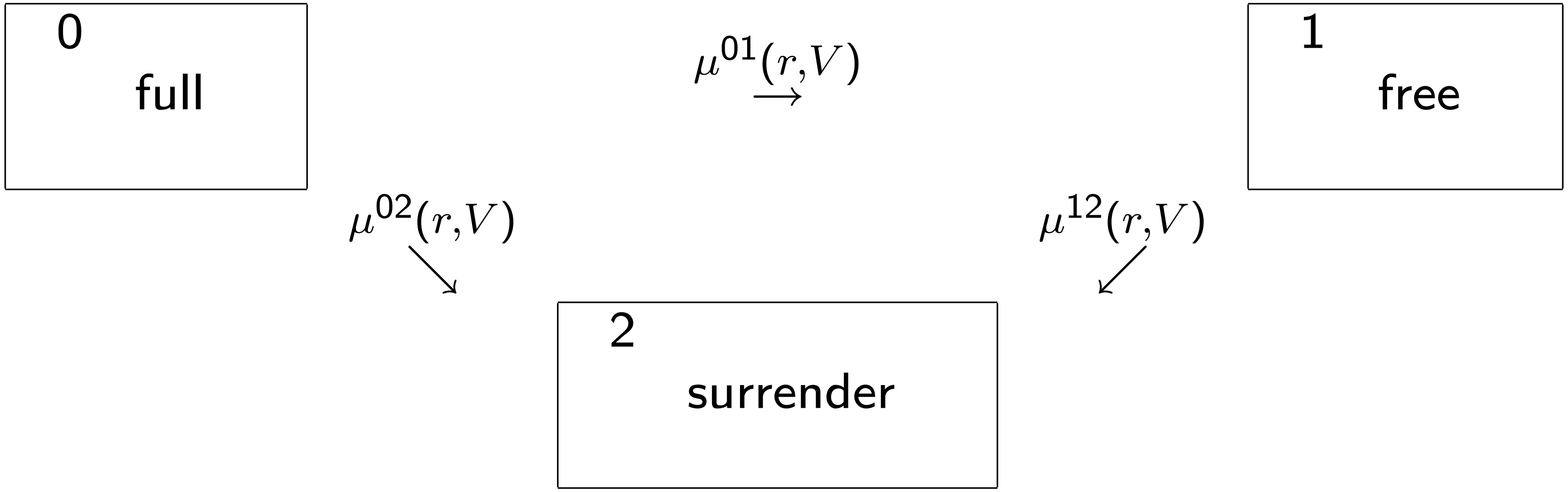

Figure 2:

Forward rates

$$e^{-\int_t^n f(t,s)ds} = E_t\left[e^{-\int_t^n r(s)ds}\right]$$

Forward mortality rates, see Milevsky and Promislow (IME 2001)

$$e^{-\int_t^n m(t,s)ds} = E_t\left[e^{-\int_t^n \mu(s)ds}\right]$$

The trouble of coexistence with forward interest rates, see Norberg (IME 2010)

$$E_t\left[e^{-\int_t^n (r(s)+\mu(s))ds}\right] \neq e^{-\int_t^n f(t,s)+m(t,s)ds}$$

Miltersen and Persson (WP 2005)

$$E_t\left[e^{-\int_t^n (r(s)+\mu(s))ds}\right] = e^{-\int_t^n f(t,s)+m^{pe}(t,s)ds}$$

$$E_t\left[e^{-\int_t^n (r(s)+\mu(s))ds}\mu(t,n)\right] = e^{-\int_t^n f(t,s)+m^{ti}(t,s)ds}m^{ti}(t,n)$$

Buchardt (IME 2014)

$$E_t\left[e^{-\int_t^n (r(s)+\mu(s))ds}\right] = e^{-\int_t^n f^i(t,s)+m(t,s)ds}$$

$$E_t\left[e^{-\int_t^n (r(s)+\mu(s))ds}\mu(t,n)\right] = e^{-\int_t^n f^i(t,s)+m(t,s)ds}m(t,n)$$

The trouble of generalization to multi-state models, see Norberg (IME 2010)

Christiansen and Niemeyer (FS 2015)

$$e^{-\int_t^n m_{jk}(t,s)ds} = E_t\left[e^{-\int_t^n \mu_{jk}(s)ds}\right]$$

Buchardt (SAJ 2017)

$$\begin{aligned} m\left(t,n\right) &= \frac{E_t\left[e^{-\int_t^n \mu(s)ds}\mu\left(t,n\right)\right]}{E_t\left[e^{-\int_t^n \mu(s)ds}\right]} \\ m_{jk}^{Z(t)}\left(t,n\right) &= \frac{E_t\left[p_{Z(t)j}\left(t,n\right)\mu_{jk}\left(t,n\right)\right]}{\underbrace{E_t\left[p_{Z(t)j}\left(t,n\right)\right]}_{q_{Z(t)j}(t,n)}} \end{aligned}$$

Buchardt, Furrer, Steffensen (arxiv 2018)

Define m_{jk} as the solution to

$$\frac{d}{ds}q_{jk}(t,s) = \sum_{l \neq k} q_{jl}(t,s)\, m_{lk}(t,s) - q_{jk}(t,s)\, m_{k\cdot}(t,s)$$

$$q_{jk}(t,s) \text{ based on KDE with } m \text{ is actually } E_t\left[p_{jk}(t,s)\right]$$

$$E_t\left[p_{jk}(t,s)\,\mu_{kl}(s)\right] \neq q_{jk}(t,s)\, m_{kl}(t,s)$$

Quite unfortunate since then

$$\begin{aligned} & E\left[\int_t^n e^{-\int_t^s r} p_{jk}(t,s)\,\mu_{kl}(s)\,ds\right] \\ \neq \; & \int_t^n e^{-\int_t^s r} q_{jk}(t,s)\, m_{kl}(s)\,ds \end{aligned}$$

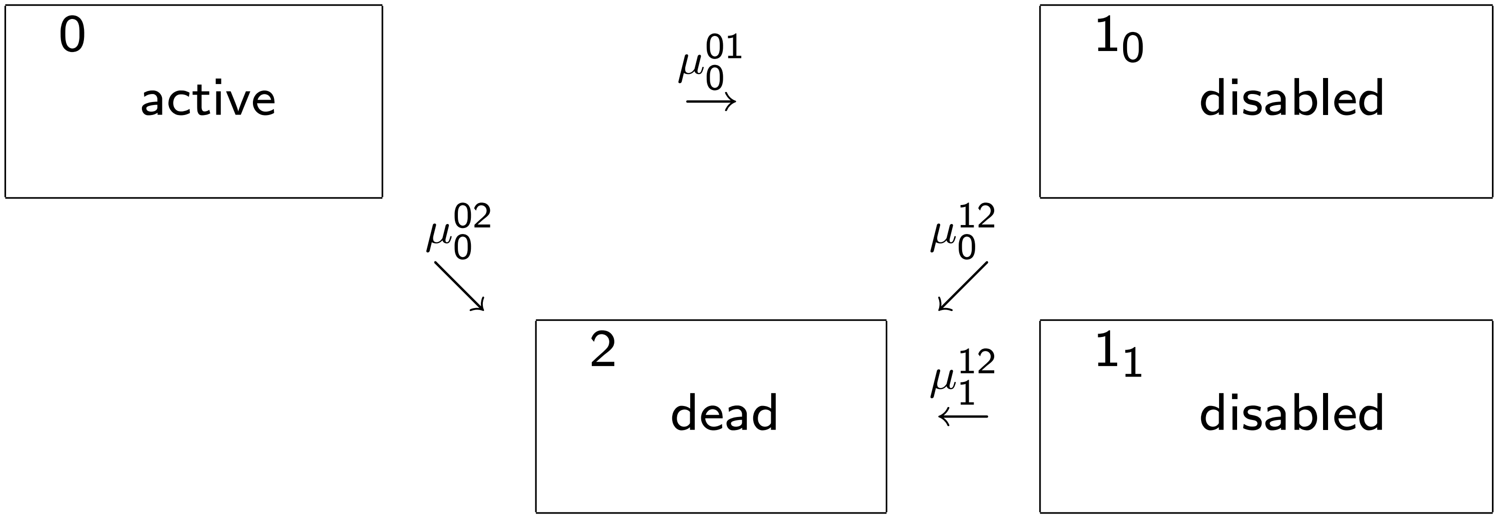
0
active
μ_0^{01}
1_0
disabled
μ_0^{02}
μ_0^{12}
2
dead
μ_1^{12}
1_1
disabled

Figure 3:

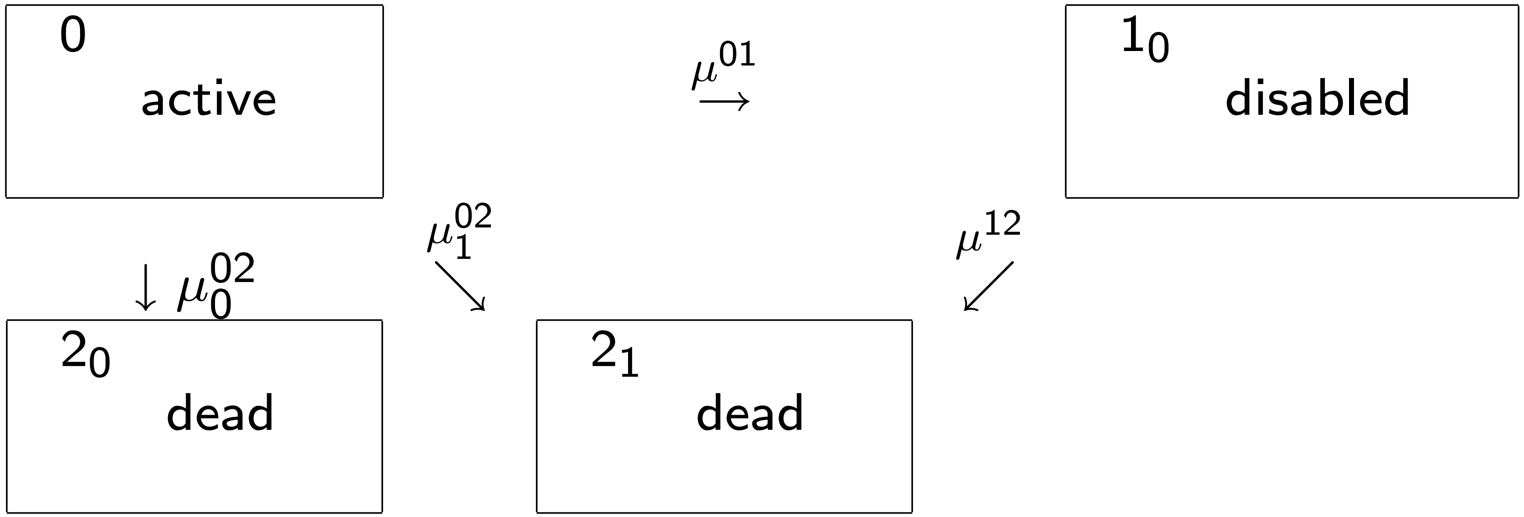

Figure 4: